Decay of a round jet

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Object

A direct numerical simulation (DNS) of a turbulent round jet is carried out on the cylindrical coordinate system to reproduce the spatial development from the formation of vortex rings to the transition to turbulence. Though a lot of careful numerical simulations of round jet have been reported recently[1]~[4], subtle discrepancy is observed in their results with one another and also with the experiments as for the point of onset of break down, the centerline velocity profile, vortical structures far downstream from the nozzle, etc.

It has been pointed out that the simulation is affected to no small extent by the coordinate system , the outflow and/or lateral boundary condition, the size of computational domain, etc. So, we are not sure as yet which simulation is the correct one as for reproduction from the formation of vortex rings to the fully developed region.

It is intended here to investigate the cause of discrepancy among the results.

Careful calculation lead us to the conclusion that the round jet is inherently too sensitive to various kinds of disturbances to follow a definitive development process unanimously reproduced.

This conclusion has been obtained by an our data-base of high accuracy and succeeding stability analysis based on it. This report describes the details of the series of our work.

1. DNS Data Base

Incompressible Navier-Stokes equation is solved numerically without using any turbulence model for round jets on the cylindrical coordinate system. The Reynolds number $Re = U_o D/\nu$, based on the nozzle diameter D and uniform velocity at the nozzle exit U_o, is 25000. Computational domain is a cylinder of length $20D$ in the mainstream direction and radius $8D$ and the number of grid points are $200 \times 80 \times 128$ in z(mainstream), r(radial), θ(circumferential) direction, respectively.

A convective boundary condition is applied at the outflow cross-section and traction-free condition[5] is employed at the lateral boundary.

Major results obtained by the simulation is shown in the following.

Figure 1 shows an instantaneous vortical structure visualized by iso-contour of total vorticity $|\omega|$, for the case of random disturbances of small amplitude superposed on the velocity prescribed profile $U(r)$ at the nozzle exit. It is confirmed that the spatial separation of vortex rings agrees with the linear stability theory.

Figure 2 compares the centerline velocity profiles of the present simulation with the experimental data by Crow & Champagne[6]. The start of reduction of the centerline velocity is farther away from the nozzle exit in the simulation than in the experiment, but in a manner more sharp.

The simulation results are reliable up to $16D$ from the nozzle exit although the computational domain covers up to $20D$, since the downstream end region may be contaminated by the reflection from the exit boundary.

Figure 3 shows the iso-contours of streamwise vorticity in a cross section at $z = 5.5D$ in the braid region between the two vortex rings. It is to be noted that 16 vortices are observed to be distributed with non-uniform separation circumferentially.

2. Linear Stability Analysis in Circumferential Direction

Since the decay of the potential core is controlled by the streamwise vortices which are found to be non-uniformly distributed as above, it is intended here to investigate the property of its growth by stability analysis.

On the basic flow $[\ 0, 0, U(r), 0\ ]$ which is the velocity profile in the braid region

$$U(r) = \frac{U_0}{2}\left\{1 - \tanh\left(\frac{1}{4}\frac{R}{\Theta}\left(\frac{r}{R} - \frac{R}{r}\right)\right)\right\}, \quad \begin{matrix} R = 0.45D \\ \Theta = 0.045D \end{matrix} \tag{1}$$

we assume superposed perturbation flow of the form

$$[\ u_r, u_\theta, u_z, p\ ] = \left[\widetilde{u_r}(r), \widetilde{u_\theta}(r), \widetilde{u_z}(r), \widetilde{P}(r)\right] e^{i(\alpha z + m\theta - \beta t)} + [\ 0, 0, U(r), 0\ ] \tag{2}$$

where i denotes the imaginary unit. After some algebra, we obtain the equation to which the perturbation pressure $\widetilde{P}\ (=\widetilde{P}_r + i\widetilde{P}_i)$ is subjected, as follows.

$$\frac{d^2\widetilde{P}}{dr^2} + \left(\frac{1}{r} - \frac{2}{U(r) - \beta/\alpha}\frac{dU}{dr}\right)\frac{d\widetilde{P}}{dr} - \left(\alpha^2 + \frac{m^2}{r^2}\right)\widetilde{P} = 0 \tag{3}$$

Eq.(3) is the characteristic Equation which has three eigen values such as $m, \alpha, \beta\ (= \beta_r + i\beta_i)$. First, we fix α, the wave number in the streamwise direction of the perturbating flow as 5.0 which is the least stable wave for $m = 0$, the axi-symmetric round jet. Then, varying m from 0 up to 40, we obtain the characteristic temporal growth rate β_i.

Figure 4 shows the the result.

A peak is found at $m = 0$, suggesting that mild distortion of circular cross section is most plausible. This distortion manifests itself in the figure(Fig.8) obtained by wavelet analysis mentioned later.

Other peaks lie over $m = 16, 23, 29, 32$, etc. However, none of these peaks are overwhelmingly dominant but have almost same growth rate. This implies that any of these mode can appear depending on the perturbation applied to simulation and also to experiments.

It is impossible to predict which mode will come true. So, we should not expect any particular process of decay in each realization of simulation and/or experiment. This explains the discrepancies of the decaying point and of vortical structures after break down, among each realization.

3. Analysis of Streamwise Vortical Structure by Wavelet Transform

In order to confirm the prediction of the above stability analysis, we examine the vortical structure in the early stage of decaying region of our simulation by means of wavelet analysis.

The continuous wavelet transform of a function $f(x)$ is defined in a form:

$$W_\psi f = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(x)\ \overline{\psi\left(\frac{x-b}{a}\right)}\ dx \tag{4}$$

where ψ stands for mother wavelet, a the scale, b the analyzing position, "——" denotes complex conjugate.

In this study, we use the simplified form of Morlet wavelet as $\psi(x)$ to analyze the local wavenumber of streamwise vortices located circumferentially:

$$\psi(x) = \cos(\sigma x)\exp(-x^2/2), \quad (\sigma = 6) \tag{5}$$

We can extract manifestation of the streamwise vortices in a cross section of each particular azimuthal separation by varying scale a.

Figures 5~8 are the examples of extracted structures demonstrated by the amplitude of the wave, in the same cross section as Fig.3. Compared with the original streamwise structure (Fig.3), it is sure that streamwise vortices consists of various wavenumber distributed non-uniformly; structures arranged at mode 16 are outstanding on the 2nd~4th quadrant, while those at mode 23 are strong on the 2nd and 4th quadrant. In addition, Fig.8 suggests a smeared $m = 3$ structure. This results indicate that even in one cross section, the manifestation of unstable mode is different azimuthally. So, we can not know which wave appears first for different realization for which infinitesimally small source of contamination may be different.

4. Conclusions

A direct numerical simulation of a turbulent round jet and its circumferential stability analysis have been carried out, and we obtained the following conclusions.

(i) Stability analysis shows that there's no dominant azimuthal mode, which implies the difficulty to predict the mode realized under a infinitesimal disturbance.

(ii) Since realized azimuthal mode is responsible for the location where the breakdown starts or vortical structures after the decay, it is possible that even the slightest difference of the condition makes the centerline velocity profile or the half-width vary.

(iii) Wavelet transform with fixed scale "a" revealed the localized streamwise vorticity and confirmed the prediction of stability analysis.

References

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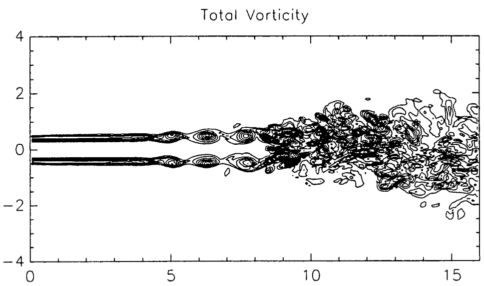

Fig.1. Contour of instantaneous total vorticity

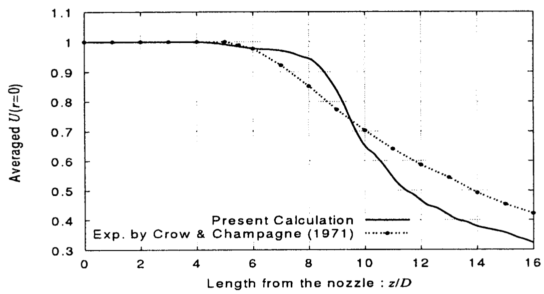

Fig.2. Centerline velocity profile

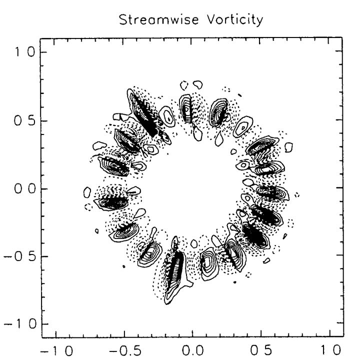

Fig.3. Contour of instantaneous streamwise vorticity on the cross-section at $z = 5.5D$

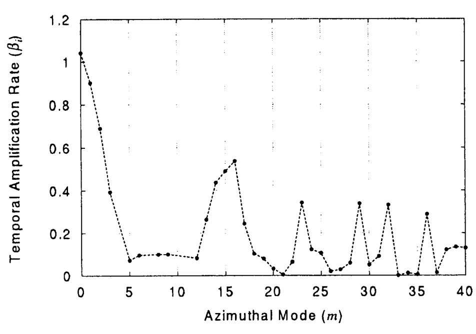

Fig.4. Temporal growth rate β_i is plotted versus the azimuthal mode m.

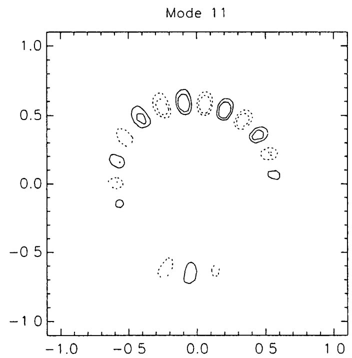

Fig.5. Contour of streamwise structure extracted by the wavelet transform at the scale corresponding to Mode 11

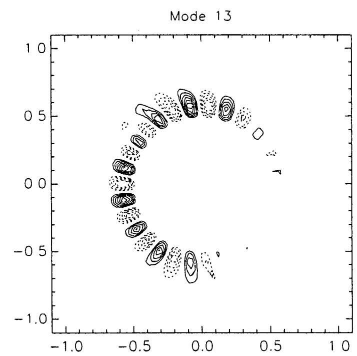

Fig.6. Mode 13

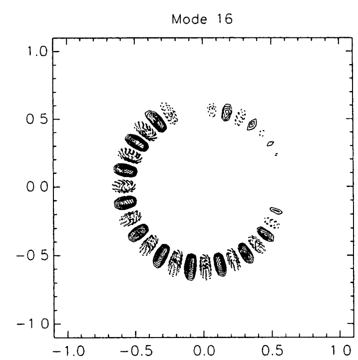

Fig.7. Mode 16

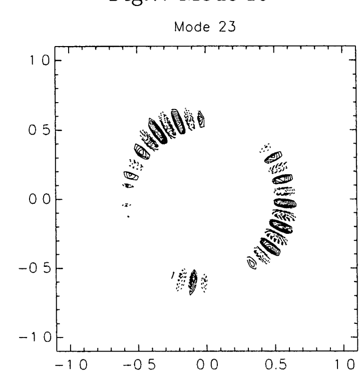

Fig.8. Mode 23