# EXPERIMENTAL STUDY OF ANNULAR SUPERSONIC MIXING LAYERS: TURBULENT KINETIC ENERGY BUDGET

**Stéphanie BELLAUD, Stéphane BARRE and Jean-Paul BONNET**
LEA/CEAT, UMR CNRS N°6609, Université de Poitiers
43 rue de l'aérodrome, 86036 POITIERS CEDEX (France)
Ph: 33-5-49-53-70-05, Fax: 33-5-49-53-70-01, E-Mail: barre@univ-poitiers.fr

## ABSTRACT

This paper presents turbulence measurements using Laser Doppler Velocimetry (LDV) in an annular supersonic mixing layer. The experimental apparatus consists in a circular supersonic jet (Dj=50mm; Mj=2.5) issuing in a subsonic flow (M≈0.2). This situation generates an annular supersonic mixing layer in which convective Mach number is close to 0.9 making possible the observation of quite strong compressibility effects. Mean and turbulent velocity fields were then obtained using 3500 to 4000 samples at each experimental point. This high number of data ensures a good statistical convergence and accuracy for LDV measurements. Mean velocity fields exhibit the usual behavior of a compressible mixing layer. The spreading rate collapses well with plane mixing layers values obtained at the same convective Mach number. The Reynolds stress tensor is also similar to the one obtained in plane mixing layer case. Then, it seems that no crucial difference exists between plane and annular flow cases when dealing with mean velocity fields and Reynolds stress tensor. Using Strong Reynolds Analogy assumptions to estimate velocity-density correlations from LDV measurements it was then possible to compute turbulent kinetic energy budget. This budget seems to be very similar to the one obtained in a compressible plane mixing layer. The present results were also compared to DNS results in the same kind of flow. Some quantitative difference between the experimental and the computed budget are observed, but a good qualitative agreement is obtained. Particularly, the dissymetry of the diffusion term is confirmed both by experiments and computations in this kind of annular flow configuration.

## INTRODUCTION

During the last decade, a lot of theoretical, computational and experimental work has been devoted to the behavior of turbulence in supersonic plane mixing layers. The main part of these studies are well reviewed in Lele (1994). It is now clear, both from computations and experiments, that the decrease of the spreading rate (when the convective Mach number Mc increases) is associated to a quite similar decrease of turbulent quantities (Barre et al. 1994, Vreman et al. 1996). These observations seem not sufficient to explain the physics of compressibility effects in order to accurately modelize them. Two important questions still remain. First, what is the real mechanism responsible for the decrease of mixing and turbulent activity at high Mach number? Second, what is the effect of geometrical conditions on the structure of turbulence in such supersonic shear layers. Some attempts were done to answer the first question. From computations, Zeman (1990) and Sarkar et al.(1991) first proposed a model based on an extra dissipation due to dilatational effects to explain the observed decrease of turbulence activity. Their computations were in quite good agreement with experimental results available at that time. With the emergence of DNS at reasonable high Reynolds number some new possible scenario for compressibility effects appeared (Sarkar 1995, Vreman et al.1996). From these DNS results it seems that the compressible dissipation remains negligible even at moderately high convective Mach numbers and that a different distribution of pressure fluctuations must be a good candidate to explain the observed decrease in turbulent activity (Freund et al. 1997). From an experimental point of view, it is now possible, with some assumptions like Reynolds analogies, to build a turbulent kinetic energy budget in a supersonic mixing layer. This kind of results are very helpful to evaluate one-point closure models and to obtain data for confrontation with subsonic budgets. Chambres et al. (1996) evaluated the turbulent kinetic energy budget in a Mc=1 plane supersonic mixing layer and found that its structure was different from those obtained experimentally (Wygnanski and Fiedler 1970, Delville 1998) and numerically (Rogers and Mosers 1994) in subsonic plane mixing layers. However, at this time, it seems that no detailed turbulent results are available in an annular supersonic configuration. That is why the present paper will then present some new experimental turbulent kinetic energy budget in an annular supersonic turbulent shear layer at high convective Mach number ($Mc \approx 0.9$). The aim of the present work is to obtain turbulent kinetic energy budget in this flow configuration and to compare it with the recent DNS results of Freund et al. (1997) concerning the turbulent kinetic energy budget in a comparable axisymmetric configuration.

## EXPERIMENTAL APPARATUS

The experiments were conducted in the S150 blowdown wind tunnel of the LEA/CEAT Poitiers. The studied flow consists in the mixing layer region of a supersonic circular jet (Dj=50mm; Mj=2.5) issuing in a subsonic flow ($M \approx 0.2$). The Reynolds number is about $2x10^8$/m for the supersonic flow. The merging of these two flows then creates an axisymmetric mixing layer. The convective Mach number is about 0.9 making possible the observation of strong compressibility effects. The boundary layers at the lips of the exit plane are fully turbulent. The momentum thickness of the supersonic boundary layer is 0.16mm, thus, the ratio of the jet radius to this thickness is $R/\theta \approx 156$. All the measurements presented in this paper were done with an Aerometrics DSA 2D LDV system. Both the supersonic and the subsonic external flows were seeded with $Si0_2$ particles less then 1 μm diameter. Average data rate of about 10kHz were obtained during the measurements and 3500 to 4000 samples were used at each experimental point to build statistics and then obtain the mean and turbulent velocity fields up to four orders moments. Before an extensive use of the LDV in this annular supersonic mixing layer, several tests must be done. We need to check the accuracy of this measurement system when applied to this sheared flow. In particular, the different measurements biases (seeding bias, velocity bias, ...) which may affect the measurements should be analyzed. Several tests have been done previously in a plane supersonic mixing layer (Mc=1) by Lammari et al. (1994), Lammari (1996) and Lammari et al. (1996) with the same LDV system. A study of convergence has been performed. Their main results are recalled here to qualify the L.D.V. system. It will be assumed, in the present paper, that the measurement and flow conditions are here sufficiently close to those of Lammari (1996). The two flows have approximatively the same physical dimensions and the same Mach and Reynolds number. Thus we will assume that Lammari's results are applicable to the present flow configuration.

### Seeding bias

The analysis of the seeding system concerns the influence of the nature of the particles and the location of the seeding. As preliminary tests, several experiments have been performed in the Mc=1 mixing layer with either $SiO_2$ particles or olive oil. In the supersonic side, a high pressure particles generator is used and the $SiO_2$ particles are introduced in the settling chamber through a blow-pipe. At contrary, in the subsonic side, the two kind of particles have been successively employed and two seeding locations have been chosen. The mean longitudinal velocity and the streamwise and transverse turbulence intensities were measured for three different seeding method in the subsonic flow. The differences between the three sets of data are negligible taking in account the experimental uncertainties. It can be concluded that the seeding particles and the location did not strongly affect the results. In a second step, a comparison between the seeding of either only the subsonic stream or only the supersonic stream or seeding both streams was done. As observed in others experiments (Debisschop 1993), some differences exist between the two « individual » seedings. It appears that the dual seeding perform a « natural » average between these two results. Without any further available and reliable theory, we choose the dual seeding for the rest of the L.D.V. experiments.

### Velocity bias

A velocity bias is introduced in L.D.V. measurements when the sampling process and the velocity magnitude are correlated. Several adjustments exist. The simpler one consists obviously in performing a new sampling of the original signal by taking in account the local flow characteristics. The goal of this method is to decorrelate the turbulent field characteristics and the sampling process (stamped sampling). It often leads to an important decrease in the effective data rate (Erdman and Tropea 1982). Thus it is necessary to collect a lot of samples and so to have a long acquisition time. In the cases of blow-down tunnel experiments, it is in general not possible to do it. So, different weighting methods can be used to take into account eventual velocity bias. Lammari had tested (Lammari (1996), Lammari *et al.* (1996) ), in the Mc=1 plane mixing layer flow, three weighting methods: the inverse of the velocity, the residence time and the inter-sample time. The results of all these tests show that no measurable velocity bias is present in the Mc=1 mixing layer. It can then be concluded that this result is still valid in the present flow because it is less sheared than the Mc=1 mixing layer studied by Lammari (1996) and Lammari et al. (1996).

### Statistic convergence

This convergence concerns the number of samples required to determine the different statistics and so to obtain the different components of the velocity and the high order moments. It is obvious that this samples number is responsible for the quality of the estimations of these moments. Several locations in the $M_c = 1$ mixing layer have been chosen to carry out the analysis of the 1st to the 4th order moments of velocity according to the sample number. The reference for the comparison is obtained with 5000 samples. It was found that the longitudinal velocity is obtained with less than 1% error with 1000 samples whereas the transverse one is obtained with less than 5% with the same number of samples. This difference is also observed for the turbulence intensities. For the Reynolds shear, a number of 3000 samples ensures a negligible error. It was observed that 3500-4000 samples are necessary to obtain the flatness coefficients with less than 1% error.

### Temporal convergence

This convergence concerns the minimum time of acquisition for the temporal average $U_T$ to be equal to $\overline{U}$ with a given precision. The characteristic time is that of large structures passage corresponding to a Strouhal number of about 0.2-0.3 (Debisschop (1993)). No specific study has been developed in the present experiment but according to different results obtained in the same kind of flow we can consider that the typical large scale structure frequency in this flow lies in the range of 5-10 kHz. This corresponds to typical characteristic time of about 100 to 200 µs. The record time in the present experiment was of the order of 0.6 s, which represents approximately 3000 to 6000 large scale structure characteristic time. We can consider that this number is sufficient to ensure a good statistic validity.

### Particles drag effects

The accuracy of the Laser Doppler Velocimetry is based on the hypothesis that the particles follow the flow. The seeding must be the more homogeneous possible and the number of the particles in the flow must be low enough to not disturb the flow but high enough to obtain a good sampling of the signal. Let's recall here that, for the present measurement, $SiO_2$ particles with a density of 2 g/cm$^3$ were used. The seeding quality and particularly the response time constant of the particles to a strong velocity gradient has been tested. Experiments on an homogeneous turbulence interacting with a normal straight shock (M=3) have been carried out (Alem 1995, Lammari 1996, Barre *et al.* 1996 ). Through the shock, the particles endure a strong decrease of velocity. The model proposed by Tedeschi (1993) which consists in an expression of the particles drag law in sliding regime, was used to process the experimental data. From this model, the mean diameter of the particles has been estimated and found equal to 0.3 µm which is consistent with the particles manufacturer data for a particles amalgamation. Thereby the time for the particles to recover the velocity of the fluid after the shock wave was found equal to 0.4 ms in the strong (M=3) shock case. It is clear that no stationary shock are present in our annular mixing layer. However, this order of magnitude is here done to show that no strong inertia effects will perturb significantly the present measurements. The mean longitudinal acceleration imposed by the shock to the particles was roughly 160x10$^6$ g corresponding to a mean velocity gradient $\partial U/\partial x$ of 4.33 10$^6$ s$^{-1}$. This value is about 150 times larger than the maximum velocity gradient expected in the present experiment. We can then conclude that the inertia effects in the mixing layer can be considered as negligible when compared to eventual other sources of experimental uncertainties. It can also be noted that, from Johnson (1989), a typical frequency response for the LDV measurements can be estimated from Stokes's drag law. This frequency, which can be considered as a -3db cutoff frequency, can be easily estimated from Johnson's formula in each kind of flow configuration. In the present case we find $f \approx 90$ kHz for the external supersonic flow and about 254 kHz for the external subsonic flow. In fact, since the particles are convected in the Lagrangian frame of reference, their capability to follow turbulent velocity fluctuations must be evaluated with large turbulent eddies convection characteristics. In the case under examination this convective velocity is about 360 m/s. At the corresponding location in the mixing layer, the characteristics frequency from Stokes's drag law is about 193 kHz which is more than thirty times the order of magnitude of typical Strouhal frequencies for the studied mixing layer.

## MEAN VELOCITY FIELD AND REYNOLDS STRESS TENSOR

LDV measurements were performed for 10 longitudinal positions between the nozzle lip and the end of the potential core. Figure N°1 shows a typical smoothed velocity profile in the asymptotic part of the flow. The lateral position Y is here normalized: $Y^*=(Y-Y_{ref})/\delta_\omega$ where $Y_{ref}$ is the position of the half-velocity point and $\delta_\omega$ the local vorticity thickness of the layer. The shape of this profile is very classical for a mixing layer. The autosimilarity is reached at about X=100mm downstream the nozzle lip. The end of the potential core is located near X=500mm downstream the nozzle(i.e. about 10 jet diameter Dj). Then this asymptotic annular mixing layer can be observed over about 400mm. We can then notice that the spreading rate of this flow is close to the one obtained in a 2D mixing layer at the same convective Mach number showing that no crucial effect of flow geometry is observed on mean velocity field compared to a 2D configuration. Reynolds stress tensor was also measured. An asymptotic state was also obtained for Reynolds stresses both in profile shape and turbulence level. Here also no significant difference is observed compared to 2D situation. As an example, figure N°2 shows the normalized longitudinal r.m.s turbulence intensity $\sigma_u/\Delta U$ as a function of $Y^*$.

## TURBULENT BUDGETS

The turbulent kinetic energy equation is written with some assumptions due to the specificities of the studied flow. The approximation of thin layer is assumed: two length scales are defined, L according to x and $\delta$ according to y with $L \gg \delta$. The mixing layer is assumed to be isobaric: $\partial \overline{P} / \partial x_i = 0$, the flow is two-dimensional in average: $\overline{W} = 0$. In this kind of supersonic flow we use the mass average (Favre) decomposition. The velocity $U$ is developed in an average velocity $\tilde{U}$ and a fluctuating one $u''$ with $\tilde{U} = \overline{\rho U} / \overline{\rho}$ and $\overline{\rho u''} = 0$. As usual, the Reynolds decomposition is used for the pressure and the density. With all the previous assumptions, the equation can be written as:

$$0 = \underbrace{\left[ -\frac{\partial}{\partial x}\left(\overline{\rho}\tilde{k}\tilde{U}\right) - \frac{\partial}{\partial y}\left(\overline{\rho}\tilde{k}\tilde{V}\right) \right]}_{\text{CONVECTION}} + \underbrace{\left[ -\overline{\rho u''v''}\frac{\partial \tilde{U}}{\partial y} \right]}_{\text{PRODUCTION}}$$

$$+ \underbrace{\left[ -\overline{\rho}\tilde{\varepsilon} \right]}_{\text{DISSIPATION}}$$

$$+ \underbrace{\left[ \frac{\partial}{\partial y}\left( \overline{\tau_{xy}u''} + \overline{\tau_{yy}v''} + \overline{\tau_{yz}w''} - \overline{p'v''} - \overline{\rho k v''} \right) \right]}_{\text{DIFFUSION}}$$

$$+ \underbrace{\left[ \overline{p'\frac{\partial u''}{\partial x}} + \overline{p'\frac{\partial v''}{\partial y}} + \overline{p'\frac{\partial w''}{\partial z}} \right]}_{\text{COMPRESSIBILITY}}$$

In the present study, no direct estimation of fluctuating densities or velocity/density correlations has been done. That is why we will use some assumptions concerning the velocity/density links. We can use the Reynolds analogies to estimate these terms. The density/velocity terms are obtained by use of the classical strong Reynolds analogy (SRA), see Gaviglio (1987) and Chambres et al. (1996). However, this analogy is not sufficient to determine all the terms of the turbulent kinetic energy equation. Thus, an instantaneous relation must be used for the triple correlations such as $\overline{\rho' u'^2}$ and $\overline{\rho' u' v'}$. This relation is called the Very Strong Reynolds Analogy (VSRA):

$$\frac{T_S'}{T_S} = -(\gamma - 1) M^2 \frac{u'}{\overline{U}} = -\frac{\rho'}{\rho}$$

By use of the VSRA we can link directly velocity and density fluctuations and then estimate turbulent energy budget from LDV measurements. The validity of the classical SRA has been extensively tested by Barre et al. (1994). They found that this analogy is valid in a mixing layer and that the velocity-density correlation coefficient remains approximatively constant ($R_{\rho u} \approx 0.9$) across a supersonic mixing layer. Concerning the instantaneous version of this analogy (VSRA), it is difficult to find accurate simultaneous velocity-density fluctuations measurements. The V.S.R.A. is in fact an extension of the S.R.A. to the instantaneous fields. It is clear that this assumption is very crude but, if we consider an adiabatic and moderately supersonic flow, the V.S.R.A. become quite acceptable when compared for example to experimental uncertainties. Using two hot wires, Smith and Smits (1993) have simultaneously estimated the velocity and temperature fluctuations in a supersonic $M_e = 2.89$ boundary layer. From their results, one can point up that in quasi-gaussian turbulences like near the center of a mixing layer, the V.S.R.A. is often at least approximately verified. Figure N°3 shows a typical kinetic energy budget for the studied flow. The terms are here normalized by $0.5(\rho_1+\rho_2)\,\Delta U^3/\delta_\omega$ where $\rho_1$ and $\rho_2$ are the external flows densities. The overall budget shape is very similar to the 2D one obtained by Chambres et al. (1996) for Mc=1. The most important result is the asymmetry of all terms, particularly the production and diffusion. This is the opposite of the subsonic case (Wygnanski and Fiedler 1970, Delville 1998) where a symmetric behavior across the mixing layer is found. Figure N°4 shows a comparison of the present data with the one obtained by Freund et al. (1997) at a similar Mach number by mean of a DNS computation. Here both results are normalized by the maximum of production term peak. Only production, diffusion and remaining terms are plotted for sake of clarity. It has to be noted that the experimental diffusion term is mainly due to lateral diffusion. The viscous and pressure diffusion terms are not measured and then are included in the remaining term. Thus, DNS budgets of figure N°4 are reconstructed from Freund et al.'s results by rearranging data in order to obtain diffusion and remaining terms featuring the same quantity as in the experimental one. It can be noticed that, even if these two budgets exhibit some discrepancies they are qualitatively comparable. The observed asymmetry of the production and diffusion terms exists in a similar way for the two sets of data. Differences between experiments and computations maybe explained by first: Convergence difficulties and low Reynolds numbers for the DNS and secondly, for experimental results, uncertainties due to LDV measurements and assumptions like SRA or VSRA to build the budgets. However the qualitative agreement between computations and experiments is quite good and leads us to assess that an important compressibility effect in this kind of flows is the asymmetry of the turbulent kinetic energy budgets at the opposite of subsonic cases where the budgets are almost symmetric.

## CONCLUSIONS

It appears, from the present results, that the structure of annular supersonic mixing layers is very close to 2D ones at the same convective Mach number if mean velocity and Reynolds stress tensor are concerned. The mean compressibility effect on the turbulent kinetic energy budgets seems to be the asymmetry of the production and diffusion terms. Despite the limited character of this experimental study including several assumptions, experimental uncertainties and limitations, it appears that a qualitative agreement between computations and experimental results is quite obtained. So, this overall agreement is encouraging and useful for the development and validation of turbulence models as well as for fundamental and applied compressible turbulence prediction and analysis.

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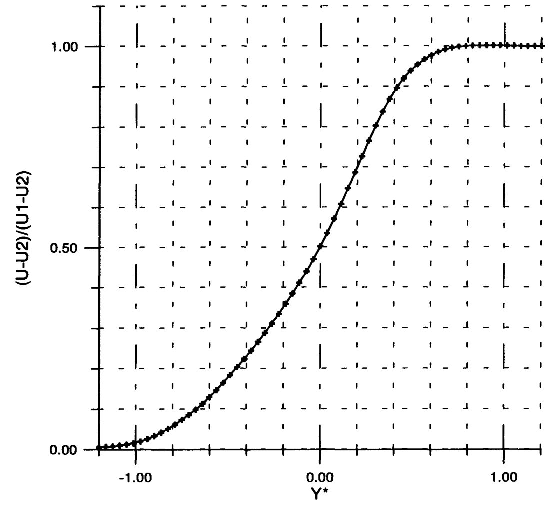

Fig.1: Mean velocity profile
For 200<X<500mm

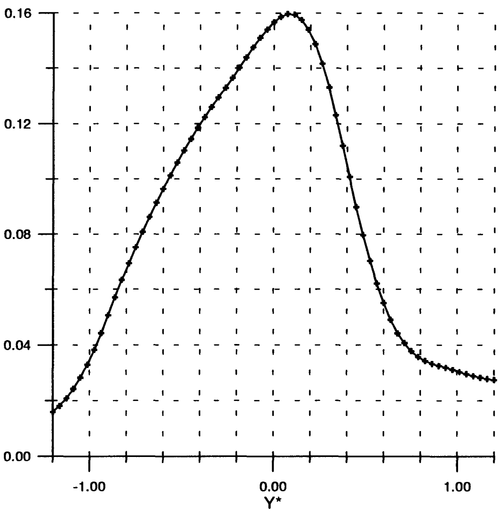

Fig2: Normalized r.m.s. longitudinal velocity fluctuations
$\sigma_u / \Delta U = f(Y^*)$

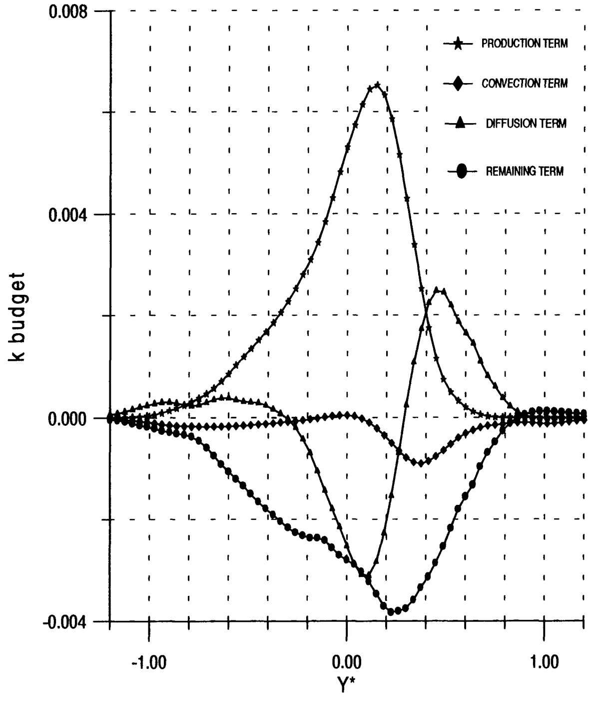

Fig. 3: Normalized experimental turbulent kinetic energy budget

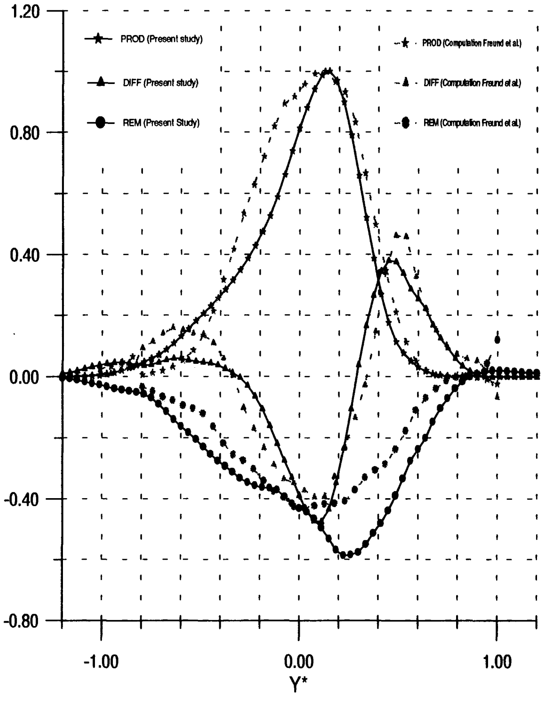

Fig. 4: Comparison between experimental and DNS turbulent kinetic energy budgets (normalized by Pmax)