EXPERIMENTAL STUDIES ON SCALING LAW PARAMETERS AND LOG—POISSON STATISTICS IN TURBULENCE

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ABSTRACT

Experimental studies on intermittent characteristics of fully developed turbulence are reported in this paper. Two independent parameters of the She—Leveque scaling law and statistical distribution of the locally averaged energy dissipation have been measured in turbulent jet. The experimental results have been compared with their theoretical values.

INTRODUCTION

Not long ago, She and Leveque (1994) have proposed a hierarchy structure model yielding the scaling laws of fully developed turbulence, which aroused a great interest recently. Dubrulle (1994) and She & Waymire (1995) have independently pointed out that the SL scaling laws imply that the statistics of the coarse-grained energy dissipation are log-Poisson. The prediction of the SL scaling laws about the scaling exponents for the velocity increment are in good agreement with experimental values up to measurable orders by Benzi et al. (1995). Chavarria et al. (1995) have made measurements in Laboratory flows to seek experimental evidence for the fundamental postulation of the She-Leveque's theory, and they have reported that there is in agreement with experimental data. However, in view of existing other opinions (Novikov, 1994; Nelkin, 1995), further experimental verification for the SL scaling laws is needed. Direct experimental evidence for the log-Poisson statistics is still lacking. In this paper, two independent parameters of scaling law: β which is characteristic of the intermittency of energy dissipation, and h which is the scale exponent of the most intermittent structure, and statistical distribution of the locally averaged energy dissipation have been measured in turbulent jet. The experimental results have been compared with their theoretical values.

She and Leveque made a fundamental postulation about the hierachy of the moments $<\epsilon_r^p>$, where ϵ_r denotes the locally averaged energy dissipation over a scale r, and $< >$ an enemble average. Specifically, they assume (She and Leveque, 1994)

$$\frac{<\epsilon_r^{p+1}>}{<\epsilon_r^p>}=A_p\left(\frac{<\epsilon_r^p>}{\langle\epsilon_r^{p-1}\rangle}\right)^\beta \epsilon_r^{(\infty)(1-\beta)}, \tag{1}$$

Where A_p are constants independent of r, and

$$\epsilon_r^{(\infty)}=\lim_{p\to\infty}\langle\epsilon_r^{p+1}\rangle/<\epsilon_r^p>. \tag{2}$$

β is a characteristic of the intermittency of energy dissipation. In the She and Levequis theory, β is a constant independent of p and takes a value 2/3. They also assume

$$\epsilon_r^{(\infty)} \propto r^{-h}, \tag{3}$$

where h is the scale exponent of the most intermittent structure, and also takes a value 2/3 in the She-Leveque's theory.

Let p=1 in (1), and using it to eliminate $\epsilon_r^{(\infty)}$, we can rewritten (1) as follows

$$\log_{10}\frac{<\epsilon_r^{p+1}><\epsilon_r>}{<\epsilon_r^p><\epsilon_r^2>}=B_p+\beta\log_{10}\frac{\langle\epsilon_r^p\rangle}{\langle\epsilon_r^{p-1}\rangle\langle\epsilon_r\rangle}. \tag{4}$$

h can be measured according to the following expression:

$$h\log_{10}r^{1-\beta}+\log_{10}(<\epsilon_r^2>/<\epsilon_r>^{1+\beta})=D_0. \tag{5}$$

She and Waymire [3]have pointed out that the SL scaling law can be exactly realized by a random multiplicative cascade process called log-Poisson. Let r_0 denotes large scale, and r small scale, then the coarse-grained energy dissipation $\epsilon_{r_0}$and ϵ_r can be related by a random mapping W_{r_0r}:

$$\epsilon_r=W_{r_0r}\epsilon_{r_0}=(r/r_0)^{-h}\beta^n\epsilon_{r_0}, \tag{6}$$

where n is a Poisson random variable with a mean λ

$$P(n)=e^{-\lambda}\lambda^n/n!,\quad n=0,1,2,\cdots \tag{7}$$

Assuming the random variables are independent, we can get

$$\langle\epsilon_r{}^p\rangle=<W_{r_0r}><\epsilon_{r_0}>=(r/r_0)^{-hp+C_0(1-\beta^p)}<\epsilon_{r_0}>, \tag{8}$$

where $C_0=-\lambda/\ln(r/r_0)$. (9)

From (8), we immediately obtain scadling exponent

$$\tau_p=-hp+C_0(1-\beta^p). \tag{10}$$

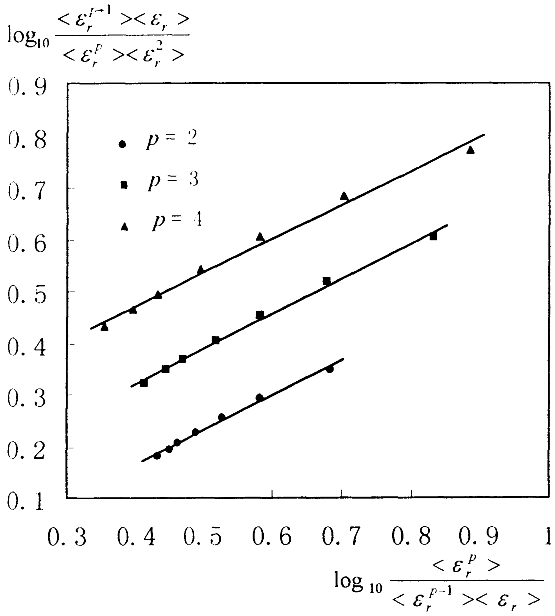
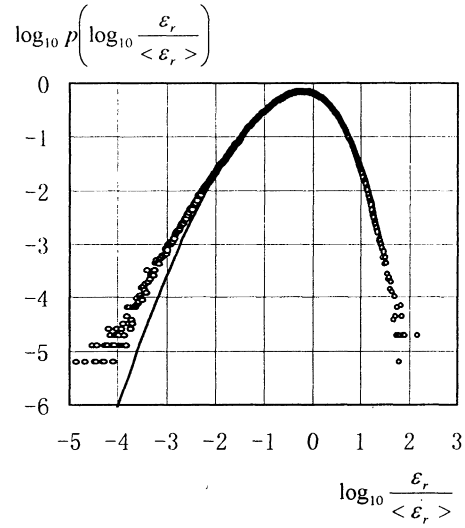


Figure 1. Measurement of the parameter β.



(a) r=30η

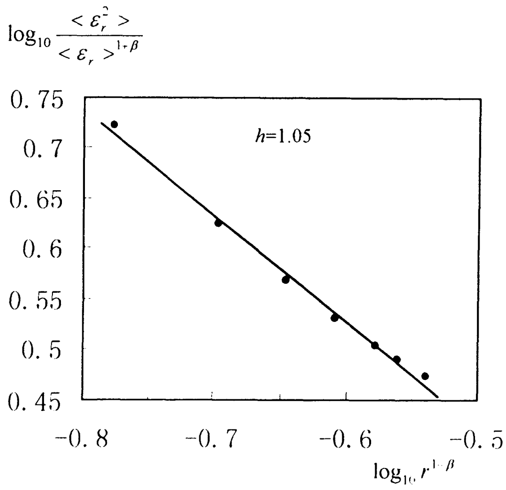
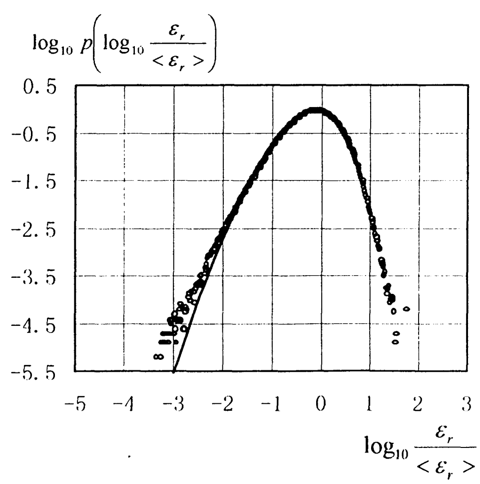


Figure 2. Measurement of the parameter h.



(b) r=80η

Figure 3. The probability density of the coarse—grained energy disspation.

By making use of the log - Poisson cascade, we can find the probability density function of small scale fluctuation from that of large scale ones. we have

$$P(\log \frac{\epsilon_r}{<\epsilon_r>}) = P(\log \frac{\beta^n}{e^{-\lambda(1-\beta)}}) \otimes P(\log \frac{\epsilon_{r_0}}{<\epsilon_{r_0}>}), \quad (11)$$

where $\otimes$ denotes a convolution. As the scale r_0 tends to the integral scale, the random variable ϵ_{r_0} tends to a constant. In this case, $P(\log(\epsilon_{r_0}/<\epsilon_{r_0}>))$ behaves approximately as a δ - function, and $P(\log \epsilon_r/<\epsilon_r>)$ will mimic closely a log - Poisson form, a smooth version of the discrete distribution function $P(\log(\beta^n/e^{-\lambda(1-\beta)}))$.

EXPERIMENTAL FACILITIES

A wind tunel with a square cross section of 60x60 cm is used to produce an air flow with a mean speed which can be adjusted between 0. 2 and 20m/s. We use a conical convergent part to reduce the wind tunnel cross section and to produce a turbulent jet. The diameter of the jet hole is 12 cm. The velocity measurements were done using a TSI1246—20WX hot film detector, which was controlled by a TSI 1050A constant temperature hot wire anemometer. The velocity probe was placed at 25 times the jet hole diameter down stream location. At this distance downstream turbulence can be considered locally homogeneous and isotropic. Time series about eight million data have been recorded for each case in order to have enough statistical accuracy. The local time measurements transformed into spatial measurements by using the Taylor hypothesis. The velocity at the jet hole was adjusted to be 17, 20, and 25 m/s. Here, we mainly use the data sets of the second case. In the other cases, we did not observe appreciable differences in the results.

RESULTS AND DISCUSSIONS

The Reynolds number based on the Taylor microscale is 864. The Kolmogorov dissipation scale η= 0. 159mm. Fixing p, and taking $r = 30\eta, 50\eta, 70\eta, 90\eta, 110\eta, 130\eta, 150\eta$ in (4), we have calculated β and B_p by making use of the least square method. The results for p=2,3,4 are shown in Fig. 1. We clearly see that the data are on straight lines in good accuracy. This illustrates that the She - Leveque's postulation (1) is good at least for low order p. The mean value of β is 0. 672, which is very near the theoretical value 2/3. Using above calculated β, and taking $r = 30\eta, 50\eta, 70\eta, 90\eta, 110\eta, 130\eta, 150\eta$ in (5), we have calculated h and D by using the least sqare method. In all cases, h varies form 1. 01 to 1. 08. This measurements contradict She - Leveque's value (2/3), but seems to support the Novikov's theory (1994) that h should equal 1.

The histograms of $\log_{10}(\epsilon_r/<\epsilon_r>)$ for $r_1 = 30\eta$ and $r_2 = 80\eta$ have been constructed from statistical samples collected in experiments. The results are shown in Fig. 3 (a) and Fig. 3 (b), where the circle represents the experimental data, and the solid line represents the theoretical curve.

We can adjust the parameter λ to fit a theoretical curve to the measured data. In case 1, we obtain λ_1= 10. 8, and in case 2, λ_2=6. 2. From (10), we have

$$C_0 = (\lambda_1 - \lambda_2)/\ln(r_2/r_1) = 4.69 \quad (12)$$

This measurement is inconsistent with She—Leveque's theoretical value $C_0 = 2$, but is close to the Chen—Cao's value C_0=4. 5 (1995).

Fig. 3(a) and (b) show that the theoretical curve fits the measured data well for large probability events, but there are some deviation for small probadility events. In the She - Leveque's theory, the n=0 event is the most intermittent event which corresponds to the right end of the theoretical curve. We can see that in both cases there are still many random events going beyond this theoretical end.

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