R_λ BEHAVIOUR OF 2ND & 4TH-ORDER MOMENTS OF VELOCITY INCREMENTS & DERIVATIVES

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ABSTRACT

The R_λ behaviour of low-order ($n = 2$ and 4) moments of velocity increments and derivatives has been examined in anisotropic shear flows over a significant R_λ range. The inertial range scaling exponents for transverse structure functions are observed to approach those of the longitudinal structure functions. Their relative behaviour suggest that any inequality may disappear at $R_\lambda \sim 5000$, depending on the degree of global anisotropy present. The $4th$-order moments of the transverse velocity derivative have approximately the same R_λ power-law exponent as the equivalent longitudinal velocity derivative moments.

INTRODUCTION & MOTIVATION

In 1941 Kolmogorov (1941, hereafter K41) published several seminal hypotheses that have shaped the course of small-scale turbulence research. A key point of these hypotheses was the assumption that the description of the small-scale structure of turbulence, at high Taylor microscale Reynolds number ($R_\lambda \equiv u'\lambda/\nu$, $\lambda = u'/\langle(\partial u/\partial x)^2\rangle^{1/2}$), could be sufficiently described by the notion of local isotropy (LI). Further attention has been afforded to the theory of small-scale turbulence since it was observed that the turbulent dissipation rate ϵ is intermittent (e.g. Kolmogorov, 1962; hereafter K62). A consequence of the intermittency of ϵ is that the nth-order moment of the longitudinal velocity increments $\langle(\delta S_L)^n\rangle$ need no longer scale as $n/3$, although exhibiting a power-law behaviour in the IR, viz. $\langle(\delta S_L)^n\rangle \sim r^{\zeta_L(n)}$, with $\zeta_L(n) \neq n/3$. Up until now, the similarity hypotheses (e.g. K62) did not distinguish between moments of longitudinal velocity increments or transverse velocity increments. Recently, experiments (Herweijer & Van de Water, 1995; Kahalerras et al., 1996) and simulations [Chen et al., 1997; Grossmann et al., 1997) show that scaling exponents associated with transverse increments may be slightly smaller than those for longitudinal increments. Alternative experiments have shown them to be significantly smaller (Antonia & Pearson, 1997; Antonia et al., 1998b). These significant differences may be explained, partly, by the choice of definition for the transverse increment and transverse scaling exponent, especially for shear flows at low to moderate R_λ. There is partial evidence (Dhruva et al., 1997; Antonia & Zhou, 1998) to indicate that the magnitudes of $\zeta_v(n)$, where v is the transverse velocity fluctuation, may increase with R_λ, leaving open the possibility that, at asymptotically large R_λ, $\zeta_u(n)$, and $\zeta_v(n)$ will be equal.

Another consequence of small-scale intermittency and RSH is that moments of Kolmogorov normalized velocity derivatives, can now be R_λ dependent. The overwhelming majority of experimental and computational data indicate that the magnitudes of the skewness $S_{\partial u/\partial x} \equiv \langle(\partial u/\partial x)^3\rangle/\langle(\partial u/\partial x)^2\rangle^{3/2}$ and flatness factor $F_{\partial u/\partial x} \equiv \langle(\partial u/\partial x)^4\rangle/\langle(\partial u/\partial x)^2\rangle^2$ of $\partial u/\partial x$ increase monotonically with R_λ (see the compilation of Sreenivasan and Antonia, 1997; hereafter SA97). An exception to this trend are the measurements of Tabeling et al. (1996) (see also Willaime et al., 1998 and references therein). In their experiment the magnitude of $F_{\partial u/\partial x}$ is observed to increase with R_λ, in a similar fashion to that compiled by SA97, except for a "transitional knee" in the region of $R_\lambda \sim 700$ to ~ 1100. In this region $F_{\partial u/\partial x}$ increases dramatically, indicating an extra increase in intermittency, before returning to values commensurate with those compiled by SA97. The corresponding behaviour for $S_{\partial u/\partial x}$ is unremarkable, the scatter in data comparable to SA97. It is speculated (Willaime et al., 1998) that the "transitional" behaviour around this "knee" is associated with the instability and breakdown of intense vortex filaments (Jimenez et al., 1993) called "worms". There is less information available for the dependence of $F_{\partial v/\partial x} \equiv \langle(\partial v/\partial x)^4\rangle/\langle(\partial v/\partial x)^2\rangle^2$ on R_λ; the limited experimental data (Antonia et al., 1996) to date suggest that for $R_\lambda \leq 200$, $F_{\partial v/\partial x}$ increases with R_λ at a rate comparable to that for $F_{\partial u/\partial x}$. The wake data of Antonia et al. (1996) indicate that the magnitude of $F_{\partial v/\partial x}$ is larger than that of $F_{\partial u/\partial x}$ but comparable to that of either $F_{\partial u/\partial y}$ or F_{ω_z}, where ω_z is the spanwise vorticity fluctuation; the significant difference between $F_{\partial u/\partial y}$ and $F_{\partial u/\partial x}$ is also evident in numerical simulations [e.g. Siggia, 1981; Boratav, 1997, Antonia et al., 1998a; Grossmann et al., 1997; Chen et al., 1997; Jimenez et al., 1993]. Siggia (1981, hereafter S81) implied that the instantaneous dissipation rate ϵ and enstrophy ω^2 may exert different influences on moments of velocity derivatives. He showed that, for homogeneous isotropic turbulence, the fourth-order velocity derivative moments can be expressed in terms of four scalar quantities (rotational invariants), I_α [α =1 to 4]. These invariants have been numerically (S81; Kerr, 1985; Gotoh & Rogallo, 1995) and experimentally (Tsinober et al., 1992; Zhou & Antonia, 1999) estimated, albeit for low R_λ flows. The numerical results, as a whole, suggest that at least two distinct exponents are needed, one for the strain and the other for the vorticity which supports the implication of S81. The experiment of Tsinober et al. (1992) indicated $I_1 - I_4$ to be closer to the corresponding Gaussian values. Zhou & Antonia (1999) measured the ratios $I_2/I_1, I_3/I_1$, and I_4/I_1 in grid turbulence; these ratios were approximately constant over the low R_λ range investigated, implying that knowledge of only one invariant is required. They concluded that this result supports the model of Phan-Thien & Antonia (1994) which proposes that the isotropic fourth-order velocity derivative tensor can be described in terms of only one invariant.

Although the longitudinal relative scaling exponent $\zeta_L(n)$ has received a great deal of attention, in the context of R_λ dependence [e.g. Arneodo et al. 1996] there has, as yet, not been an equivalent investigation for $\zeta_T(n)$. A principal aim of this paper is to conduct such an investigation for $\zeta_T(n)$ [n = 2, 4] over a relatively wide R_λ range [$\sim$40 to $\sim$ 1400]. Of interest is the equality $\zeta_T(n) = \zeta_L(n)$ - either it is not valid or it may become valid at high R_λ. A secondary aim is to conduct an investigation of the R_λ dependence of $F_{\partial v/\partial x}$ in the spirit of SA97 and Willaime et al. (1998). It is of interest to know whether $F_{\partial v/\partial x}$ increases monotonically with R_λ, as reported by SA97 for $F_{\partial u/\partial x}$, or whether it exhibits a "transition", as reported by Willaime et al. (1998). The low R_λ experimental and DNS results suggest that transverse gradients are more intermittent than longitudinal gradients. It of interest to establish if the relative rates of increase with R_λ for $F_{\partial u/\partial x}$ and $F_{\partial v/\partial x}$, in the context of S81, are different.

EXPERIMENTAL METHODS

A significant amount of data from a number of different flows over a wide range of R_λ ($40 \lesssim R_\lambda \lesssim 1400$) was analyzed for this paper. The following flows were used: cylinder wakes $R_\lambda \sim 38$ and $R_\lambda \sim 210$ (Antonia & Pearson, 1997); pipe - single-wire ($67 \lesssim R_\lambda \lesssim 337$); X-wire data ($68 \lesssim R_\lambda \lesssim 335$) (Pearson, 1999);plane jets — single-wire ($500 \lesssim R_\lambda \lesssim 1400$), X-wire data $R_\lambda \sim 535$ (Antonia et al., 1997b), $666 \lesssim R_\lambda \lesssim 1170$ (Pearson, 1999). In each flow, a crossed hot-wire probe was used to measure u and v velocity fluctuations. Other than the following brief comments, a detailed description of each experiment can be found in the cited references. Also, we have recently measured spatial transverse increment on the centreline of a plane jet at high $R_\lambda(\sim 1000)$ using parallel wires (Zhou et al., 1999). Data were acquired with in-house constant temperature anemometers at an overheat ratio of 1.5. Signal conditioning was achieved with an in-house amplifier/low-pass filter (24 dB/octave) combination. We make use of Taylor's hypothesis to convert time to space. Most of the data have high values of $u'/\langle U\rangle$ and should be corrected in the high wavenumber/small-scale range. We have not applied any corrections - except for the technique discussed in the section dealing with derivatives. We estimated the number of independent samples, $N \equiv t_r/2T_u$ (Tennekes & Lumley, 1972) and consider it to be indicative of the worst-case scenario. Here t_r is the total record length time and T_u is the integral time-scale [$\equiv \int_0^{\tau_0} \rho_{uu}(\tau)d\tau$; where $\rho_{uu}(\tau)$ is the longitudinal velocity autocorrelation function and τ_0 is the time at which the first zero crossing occurs]. Typically, between 15000 and 45000 independent longitudinal samples were obtained for the flows.

RELATIVE SCALING EXPONENTS

Convincing scale invariance for nth-order moments of δS_L has yet to be observed in experiments (e.g. Anselmet et al., 1984), possibly due to a number of reasons, e.g. lack of such spatial homogeneity, insufficient R_λ, mean shear and global anisotropy. Therefore, we will only consider relative scaling exponents estimated using ESS (Benzi et al., 1993). For this study, $\zeta_L(n)$ is estimated, with u and r in the direction of the mean velocity [i.e. $\delta S_L = u(x+r) - u(x)$], by $\left\langle |(\delta S_L)^a|^{n/a}\right\rangle \sim \langle|(\delta S_L)^3|\rangle^{\zeta_L(n)}$ in the range where $\langle(\delta S_L)^3\rangle$ best shows approximate linearity with r. The first ζ_T to be considered, $\zeta_{T,1}(n)$ (e.g. Antonia & Pearson, 1997; Antonia et al., 1998b), is a relative comparison to δS_L viz.$\left\langle |(\delta S_T)^a|^{n/a}\right\rangle \sim \langle|(\delta S_L)^3|\rangle^{\zeta_{T,1}(n)}$, i.e. both δS_L and δS_T are temporal and measured with the $X - wire$ probe, e.g. $\delta S_T \equiv v(x+r) - v(x)$. The second, is $\zeta_{T,2}(n)$ viz.$\left\langle |(\delta S_T)^a|^{n/a}\right\rangle \sim \langle|(\delta S_T)^3|\rangle^{\zeta_{T,2}(n)}$,which, while having less theoretical justification than $\zeta_{T,1}(n)$, has been previously reported (Herweijer & van de Water, 1995; Kahalerras et al., 1996) for spatial δS_T, e.g. $u(z+r) - u(z)$. A new type of transverse scaling component is investigated, e.g. $\zeta_{T,3}(n)$, viz. $\left\langle |(\delta S_L)^a(\delta S_T)^{2a}|^{n/3a}\right\rangle \sim \left\langle |(\delta S_L)^a(\delta S_T)^{2a}|\right\rangle^{\zeta_{T,3}(n)}$. For isotropic turbulence the moment $\langle\delta S_L(\delta S_T)^2\rangle$ should also scale like r (Monin & Yaglom, 1975). Lastly, $\zeta_{T,4}(n)$ is a spatial one, viz. $\left\langle |(\delta S_T)^a|^{n/a}\right\rangle \sim \langle|(\delta S_L)^3|\rangle^{\zeta_{T,4}(n)}$ where $\delta S_T \equiv u(z+r) - u(z)$. To date we only have one $R_\lambda(\sim 1000)$ result available (Zhou et al., 1999).

To estimate any $\zeta(n)$ from ESS type cross-plots a linear regression curve fit is required to log-log data. There is uncertainty $e_n(r)$ associated with both the abscissa and ordinate variables and these uncertainties must be accounted for in the linear regression method. We use a modified result of Benedict and Gould (1996). They proposed a general formula for the variance of a centered stochastic variable, γ, with unknown distribution to be, for the nth-moment, e.g. $var_\mu(n) = \gamma_{2n} - (\gamma_n)^2 + (n\gamma_{n-1})^2\gamma_{2n+1} - 2n\gamma_{n+1}\gamma_{n-1}$. We have ignored the fact that results for odd-order moments are slightly inaccurate as the variables $|\delta\beta(r)|$ (where β is either u or v) are not centered. Note that this method requires knowledge of the higher-order moment; in practice we must tolerate some non-closure. With the variance calculated for all values of $\langle|\delta\beta(r)|^n\rangle$, it is now possible to fit power-laws to cross-plots of $\langle|\delta\beta(r)|^n\rangle$ versus $\langle|\delta\beta(r)|^3\rangle$. Note however that we must take into account the associated variance for each point in both the abscissa and ordinate directions. There are many methods to use — a good outline of the topic is given in Press et al. (1994). From Press et al., we have chosen to use a χ^2 minimization scheme (their routine *fitexy*) although it assumes the variance to be normally distributed — a small and insignificant error considering that the final uncertainty in ζ is mostly dependent on N.

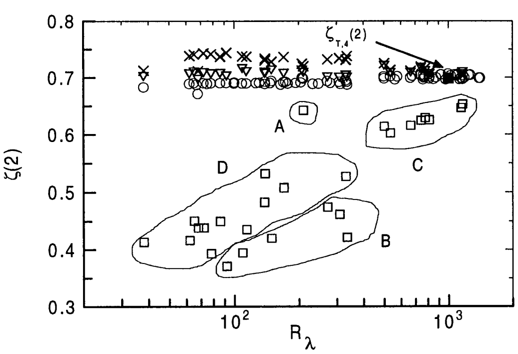

Figure 1. R_λ dependence of scaling exponents for $\langle(\delta u)^2\rangle$, $\langle(\delta v)^2\rangle$ and $\langle|\delta u(\delta v)^2|^{2/3}\rangle$ for $R_\lambda \sim 40 - 1400$. In Figures 1 and 2 both single & X-wire data is used for $\zeta_L(n)$. Regions marked A, B, C and D, are discussed in the text. Uncertainty bars are not shown for clarity. ○, $\zeta_L(2)$; □, $\zeta_{T,1}(2)$; ×, $\zeta_{T,2}(2)$; ▽, $\zeta_{T,3}(2)$; ■, $\zeta_{T,4}(2)$ (a single point indicated by the arrow).

Figures 1 and 2 show the R_λ dependence of the five scaling exponents for $n = 2$ and 4, respectively. Typical uncertainties range from ~ 0.017 to 0.025 for $\zeta(2)$ and 0.033 to 0.05 for $\zeta(4)$. ζ_L does show a slight R_λ dependence and this trend is upward increasing for $n = 2$ and downward decreasing for $n = 4$ and since cross-plotting is relative to $n = 3$ any trend must change sign as n crosses 3. The result for $\zeta_{T,1}(n)$ is very R_λ dependent and is consistent with previous reports (e.g. Antonia et al., 1998b). The high magnitude of the inequality $\zeta_{T,1}(n) < \zeta_L(n)$, at least for small to moderate R_λ, is most probably due to the lingering effect of global anisotropy, α ($\equiv \langle v^2\rangle / \langle u^2\rangle$). Since $\langle|(\delta v)^a|^{n/a}\rangle$ is cross-plotted against $\langle|(\delta u)^3|\rangle$ — a different variable — there is no change in sign for the R_λ dependence trend from $n < 3$ to $n > 3$ as there is for $\zeta_L(n)$, $\zeta_{T,2}(n)$ and $\zeta_{T,3}(n)$. The existence of the inequality is not surprising since we are estimating $\zeta_{T,1}(n)$ from a cross-plot of two different variables, δu and δv, with different longitudinal and transverse integral length scales (i.e. $L_u > 2L_v$). As α becomes smaller than 1, L_v falls below its isotropic value of $L_u/2$. This reduction results in $\langle|\delta v|^n\rangle$ becoming more quickly decorrelated than $\langle|\delta u|^n\rangle$ — the result is a continual reduction in the local slope as r decreases and an erosion of any IR. However, this effect is reduced, as is the inequality $\zeta_{T,1}(n) < \zeta_L(n)$, as R_λ increases and the overwhelming consequences are an increase in the extent of the IR and a reduced influence of large-scale anisotropy as scales reduce in size. Some "groups" of $\zeta_{T,1}(n)$ results shown in Figures 2 and 3 are worth highlighting. The two high values, marked "A" at $R_\lambda \sim 205$ are the results for the high-speed cylinder wake (i.e. $u - v$ and $u - w$ centreline measurements). For these two runs, α is equal to 0.88 and 0.89 respectively. Conversely, the low group of seven runs marked "B" are the off-centreline measurements for the pipe investigation. Within this group, α ranged from 0.36 to 0.69 and S^* ranged from 0.06 to 0.077, the increased anisotropy results in a dramatic increase in the inequality $\zeta_{T,1}(n) < \zeta_L(n)$. The majority of the data (within areas marked C and D) are centreline measurements and appear to follow a plausible R_λ dependence considering that α is approximately constant ($0.71 \le \alpha \le 0.77$). It is not implausible that the R_λ trend implies a steady reduction in the inequality $\zeta_{T,1}(n) < \zeta_L(n)$, which may be minimized at $R_\lambda \sim 5000$.

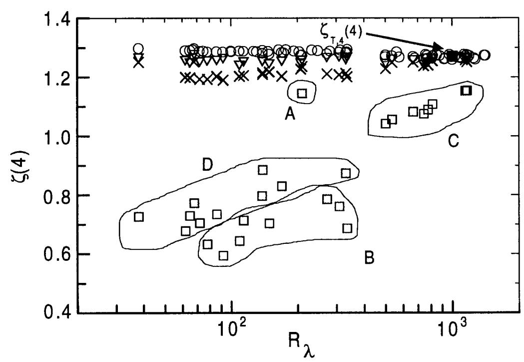

Figure 2. R_λ dependence of scaling exponents for $\langle(\delta S_L)^4\rangle$, $\langle(\delta S_T)^4\rangle$ and $\langle|\delta S_L(\delta S_T)^2|^{4/3}\rangle$.Regions marked A, B, C and D, are discussed in the text. Uncertainty bars are not shown for clarity. ○, $\zeta_L(4)$; □, $\zeta_{T,1}(4)$; ×, $\zeta_{T,2}(4)$; ▽, $\zeta_{T,3}(4)$; ■, $\zeta_{T,4}(4)$.

We note that the R_λ dependence of $\zeta_{T,2}(n)$ is not unlike the results of Kahalerras et al. (1996) and Herweijer & Van de Water(1995). The values of $\zeta_{T,2}(n)$, in our case, are obtained by cross-plotting against the same component, i.e. $\delta S_T \equiv v(x + r) - v(x)$. Again, a R_λ dependence is evident with a sign change between $n < 3$ and $n > 3$. An inequality between

$\zeta_{T,2}(n)$ and $\zeta_L(n)$ exists but it is smaller than that between $\zeta_{T,1}(n)$ and $\zeta_L(n)$ and this fact is not surprising since the cross-plotting of like-variables emphasizes the self-similarity of the *nth*-moment relative to the third-moment because their origin is the modulus of a common pdf. Any connection with turbulence theory is tenuous. The results for $\zeta_{T,3}(n)$, viz. $\langle|(\delta S_L)^a(\delta S_T)^{2a}|^{n/3a}\rangle \sim \langle|\delta S_L \delta S_T^2|\rangle^{\zeta_{T,3}(n)}$ appear to be a balance between $\zeta_L(n)$ and $\zeta_{T,2}(n,)$ which is the only exponent that does not show any R_λ dependence in Figures 1 and 2. Of current interest is the striking result for $\zeta_{T,4}(n)$. There is no difference between the definition of $\zeta_{T,4}(n)$ and $\zeta_{T,1}(n)$, other than the use of spatial instead of temporal increments for δS_T. Associated with this fact is the measurement technique used to acquire δS_T. However, it is expected, for LI, that there should be no difference in the inequalities between $\zeta_{T,4}(n)$ and $\zeta_L(n)$ or $\zeta_{T,1}(n)$ and $\zeta_L(n)$. Figures 1 and 2 clearly show that a difference does exist, and this fact may be indicative of the effect of α on IR separations. It is naive to believe that, in anisotropic flows, all measurements of $\zeta_T(n)$, whether by different technique or definition, should result in a unique globally "homogeneous" value. It is conjectured that only an increase in R_λ, with the corresponding increase in separation between dissipative and integral length scales, could rectify this anomaly.

4th-ORDER MOMENTS OF DERIVATIVES IN TURBULENT FLOWS

The use of one $X-probe$ will yield, with the use of Taylor's hypothesis, the three independent components of the fourth-order velocity gradient tensor, i.e. $F_1 = \left\langle(\partial u/\partial x)^4\right\rangle$; $F_2 = \left\langle(\partial u/\partial x)^2(\partial v/\partial x)^2\right\rangle$ and $F_3 = \left\langle(\partial v/\partial x)^4\right\rangle$ and these three parameters are related to I_α as $F_1 = 4I_1/105$, $F_2 = I_1/105 + I_2/70 - I_3/105$ and $F_3 = 3I_1/140 + 11I_2/140 - 3I_3/35 + I_4/80$. Here, we present results for F_1 to F_3, measured with a $X-wire$ probe over a large R_λ range ($40 \lesssim R_\lambda \lesssim 1400$). Note that pdf methods are susceptible to anemometer noise and probe response/resolution limitations. The anemometer noise is easily accounted for with low-pass filtering. The probe response/resolution correction for pdfs has yet to be fully studied. Our crude estimation method utilizes $\langle(\delta\alpha)^n\rangle$ to estimate $\langle(\partial\alpha/\partial x)^n\rangle$ viz. $\langle(\partial\alpha/\partial x)^n\rangle \sim \lim_{r\to 0}\langle(\delta\alpha)^n\rangle/r^n$. We ignore all separations, r, below some r which we assume corresponds to the maximum wavenumber, k_1, after which the compensated spectrum, $k_1^{5/3}\phi(k_1)$, deviates from an exponential behaviour. We use a band of r, that corresponds to the band of k_1 which clearly follows $k_1^{5/3}\phi(k_1)$, for extrapolation to $r = 0$. This band, which now consists of spatial scales r using Taylor's hypothesis, is indicated in Figure 3 between the single and double arrows. A cubic spline is fit between these two r scales using a function of the type $\ln[\langle(\delta\alpha)^n\rangle/r^n] = g(r^3)$; this is shown in Figure 3 as hatched ($\times$) line in the region between the single and double arrows. The resulting cubic splines are used as weighting functions to extrapolate high-order polynomials of the type $\ln[\langle(\delta\alpha)^n\rangle/r^n] = g(a_0^j + \sum_{i=1}^{m} a_i^j r^i)$ to $r = 0$ (the superscript j indicating the j^{th} polynomial) — without the weighting provided by the cubic splines, only low-order polynomials would be possible. The maximum order of m is usually 9. The estimates for $\langle(\partial\alpha/\partial x)^n\rangle^j$ i.e. $\exp(a_0^j)$, for $1 \le j \le 8$ are statistically averaged and the mean and the $\pm 95\%$ confidence levels are calculated. Values of $\exp(a_0^j)$ that are not within the $\pm 95\%$ limits are discarded and the process is repeated until all values fall within the $\pm 95\%$ limits. This usually results in the eight estimates for $\langle(\partial\alpha/\partial x)^n\rangle^j$ being reduced to three or four. The mean of these three or four values are then averaged with the estimate for $\langle(\partial\alpha/\partial x)^n\rangle$ from the first method and corresponding uncertainties are added.

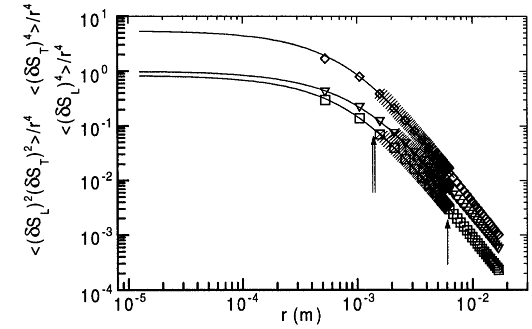

Figure 3. r^{-4} compensated fourth-order moments of $\langle(\delta S_L)^4\rangle$, $\langle(\delta S_T)^4\rangle$ and $\langle(\delta S_L)^2(\delta S_T)^2\rangle$ in terms of r. Each curve is "post"-normalized by the resulting $\langle(\delta S_L)^4\rangle/r^4|_{r=0}$. The $\uparrow$ and $\Uparrow$ indicate the start and finish of the extrapolation weighting region. ∇, $\langle(\delta S_L)^4\rangle/r^4$; $\diamond$, $\langle(\delta S_T)^4\rangle/r^4$; $\square$, $\langle(\delta S_L)^2(\delta S_T)^2\rangle/r^4$; $\times$, spline fit to $\langle(\delta S_L)^4\rangle/r^4$ or $\langle(\delta S_T)^4\rangle/r^4$ or $\langle(\delta S_L)^2(\delta S_T)^2\rangle/r^4$; —, "averaged" extrapolation to $r = 0$.

The R_λ dependence of $F_{\partial u/\partial x}$ and $F_{\partial v/\partial x}$ is shown in Figure 4; for completeness, results for $F_M \equiv \langle(\partial u/\partial x)^2(\partial v/\partial x)^2\rangle/\langle(\partial u/\partial x)^2\rangle\langle(\partial v/\partial x)^2\rangle$, inferred from the limiting values of $r^{*-4}\langle(\partial u)^2(\partial v)^2\rangle$, have been included. All three quantities appear to increase with R_λ at approximately the same rate (the power-law exponents being 0.27 ± 0.05; 0.30 ± 0.08 and 0.29 ± 0.06, respectively). In particular, there is no indication of a "transitional" behaviour at $R_\lambda \simeq 700$, as observed by Tabeling et al. (1996). Similarly, Figure 5 shows that, the ratios $F_3/F_1 \equiv \langle(\partial v/\partial x)^4\rangle/\langle(\partial u/\partial x)^4\rangle$ and $F_2/F_1 \equiv \langle(\partial u/\partial x)^2(\partial v/\partial x)^2\rangle/\langle(\partial u/\partial x)^4\rangle$ are approximately independent of R_λ (here, the power-law exponents are -0.03 $\pm$ 0.04 and -0.04 $\pm$ 0.03, respectively). This result consolidates the earlier suggestion (Antonia et al., 1996), based on a limited R_λ range ($R_\lambda \lesssim 200$), that $F_{\partial v/\partial x}$ increases with R_λ at a rate comparable to that for $F_{\partial u/\partial x}$. The present averaged values of F_3/F_1 and F_2/F_1, assuming R_λ independence, are close to about 5.52 and 0.84. The power-law exponents for $F_{\partial u/\partial x}$ and $F_{\partial v/\partial x}$ are in reasonable agreement with the model of Pullin & Saffmann (1993; $n = 0.25$) based on the Lundgren-Townsend vortex model for small-scale turbulence. This model predicts that all velocity derivative moments increase at an equal rate. Indeed, Pullin

& Saffmann also predict $F_3/F_1 = 6$, independently of R_λ, which is in good agreement with the current experimental results, considering the difficulty of acquiring adequately resolved fine-scale measurements. Also included in Figure 4 are DNS results. The DNSs [e.g. S81[1]; Boratav[2], 1998, Antonia et al., 1998a[3]; Grossmann et al.[4], 1997; Chen et al.[5], 1997; Jimenez et al.[6], 1993] were all completed under different conditions, e.g. stationary/non-stationary, forced/unforced and full Navier-Stokes/high-symmetry assumption, and irrespectively of this, all tend to approximately the same R_λ independent value. Although the current results show that F_1, F_2 and F_3 increase with R_λ at approximately the same rate, and, as a consequence, F_3/F_1 and F_2/F_1 are constant with magnitudes comparable to the DNS values, no conclusion concerning the invariants $I_1 - I_4$ can be made as only a single $X-wire$ probe was used and the three relations describing F_1, F_2 and F_3, in terms of the four unknowns $I_1 - I_4$ are, therefore, indeterminate. Added to this dilemma is that DNS results show that F_3/F_1 and F_2/F_1 can be constant (with magnitudes in agreement with the current experimental results) while $I_1 - I_4$ increase at different rates while the experimental results of Zhou & Antonia (1998) also show F_3/F_1 (~ 4) and F_2/F_1 ($\sim 2/3$) to be constant (with Gaussian magnitudes, in disagreement with the current results) while $I_1 - I_4$ increase at the same rates. Also shown in Figure 5 are $(F_3/F_1)'$ ($\sim 3.98 \pm 0.31$) and $(F_2/F_1)'$ ($\sim 0.69 \pm 0.04$), the prime indicating estimation using forward differencing. The average values of $(F_3/F_1)'$ and $(F_2/F_1)'$ are equal to that predicted by Phan-Thien & Antonia (1994) and agree with the experimental results of Zhou & Antonia (1998), yet all the current experiments could not resolve the Kolmogorov dissipative scale and required the extrapolation technique described above to estimate F_3/F_1 and F_2/F_1. This may explain the results of Tsinober et al. (1992) and Zhou & Antonia (1998), i.e. the use of forward differencing with a period equivalent to the sampling frequency may result in moments close to Gaussian, perhaps due to poor probe resolution and response.

[1]Siggia quoted the results for the four invariants I_1, I_2, I_3 and I_4.

[2]Boratav quotes the invariants of Siggia, for his simulation, to be $I_1 \sim 2.5605$, $I_2 \sim 4.2764$, $I_3 \sim 0.6121$, and $I_4 \sim 17.8838$

[3]The values of T and M for these simulations were calculated from the $4th-order$ structure functions i.e. the same method used for experimental data.

[4]assuming $\langle(\partial v/\partial x)^2\rangle = 2\langle(\partial u/\partial x)^2\rangle$.

[5]Estimated from their Figure 1 and assuming $\langle(\partial v/\partial x)^2\rangle = 2\langle(\partial u/\partial x)^2\rangle$.

[6]Estimated from Table V of Zhou and Speziale (1998) and assuming $\langle(\partial u/\partial y)^2\rangle = 2\langle(\partial u/\partial x)^2\rangle$.

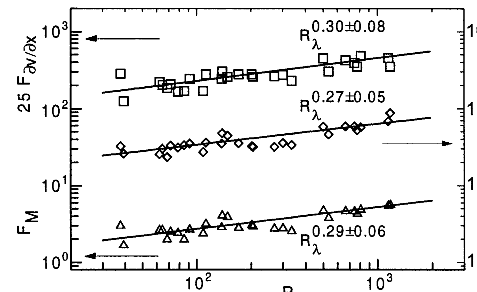

Figure 4. R_λ dependence of velocity derivative flatness factors. $\diamond$, $F_{\partial u/\partial x}$; $\square$, $F_{\partial v/\partial x}$; $\triangle$, F_M. ——, χ^2 power-law fit (Press et al., 1994); the resulting power-law exponents are shown. Uncertainty bars are not shown for clarity.

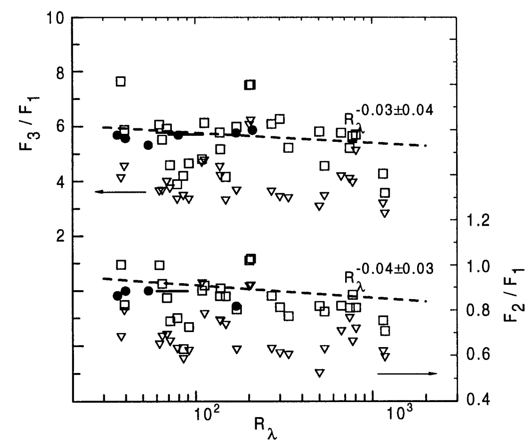

Figure 5. R_λ dependence of the moment ratios F_3/F_1 and F_2/F_1. Uncertainty bars are not shown for clarity. $\square$, a selection of $X-wire$ velocity measurements of the current study; ∇, applying forward differences to the same data; • and — — , various DNS data [e.g. S81; Boratav, 1997, Antonia et al., 1998a; Grossmann et al., 1997; Chen et al., 1997; Jimenez et al., 1993]

CONCLUSIONS

Longitudinal and transverse velocity structures fucntions have been measured in a variety of turbulent shear flows over a reasonable R_λ (40-1400) range. The transverse inertial range scaling exponent has been defined a number of different ways. The inequalities between the longitudinal and any of the transverse inertial range scaling exponents decrease with R_λ, suggesting equality at $R_\lambda \sim 5000$. The flatness factors for both longitudinal and transverse velocity derivatives have been estimated from their equivalent nth -order increment moments in the limit of small r. The R_λ power-law dependence for $F_{\partial u/\partial x}$, $F_{\partial v/\partial x}$ and F_M are all, within experimental uncertainty, approximately the same. The ratio $\langle(\partial v/\partial x)^4\rangle / \langle(\partial u/\partial x)^4\rangle$ is approximately constant, its magnitude (~ 5.52) comparable to that from DNS, and not too dissimilar to that predicted by Pullin & Saffmann (6).

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