NUMERICAL SIMULATION OF TRANSONIC BUFFET FLOWS USING VARIOUS TURBULENCE CLOSURES

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ABSTRACT

Numerical simulation of buffet flows have been performed using various turbulence models, including linear and non-linear low-Re eddy-viscosity models. The accuracy of the models has been assessed against experimental data for transonic flows around the NACA-0012 aerofoil. Discretisation of the mean flow and turbulence transport equations is obtained by a Riemann solver up to third-order accurate. The time integration is obtained by an implicit unfactored method. The study shows that two-equation models associated with functional c_μ coefficient in the calculation of the eddy-viscosity (henceforth labelled NL-c_μ), provide better results. The Spalart-Allmaras one-equation model provides comparable results with the above models, while larger inaccuracies are presented in the case of linear and non-linear models with constant c_μ coefficient. The buffet onset boundaries are similarly predicted by the one-equation and NL-c_μ models.

INTRODUCTION

A significant effort to validate turbulence models in steady aerodynamic flows has been spent over the past decade (eg Haase *et al.*, 1993; Leschziner 1998, Loyau et al. 1998, Barakos and Drikakis, 1997a). However, much less information has been accumulated in connection with the validation of turbulence models in unsteady aerodynamic flows featuring buffet and/or dynamic-stall. Concerning dynamic-stall, recent studies have been performed (Barakos and Drikakis 1997b, 1999; Barakos *et al.*1998) using a variety of low-Re linear and non-linear eddy-viscosity models (EVM). These studies revealed that non-linear EVMs can indeed offer better accuracy than algebraic and one-equation models, in predicting dynamic-stall both in subsonic and transonic flows over pitching and oscillating aerofoils. On the other hand, buffet computations have so far been performed by using, mainly, algebraic turbulence models (Edwards, 1996; Girondroux and LeBalleur, 1988). Therefore, the present study has been initiated to assess more advanced closures in transonic flows around aerofoils featuring buffet.

Transonic buffet appears in many aeronautical applications such as internal flows in compressor passages, around turbomachinery blades as well as in external flows over aircraft wings. The aerodynamic performance in these applications depends strongly on the unsteady shock/boundary-layer interaction which may change position around the aerofoil due to the self-excited shock oscillations. Accurate prediction of such flow phenomena is of significant technological importance and their simulation remains a challenging problem due to the complex physics involved.

Past research has revealed that the accuracy of the numerical calculations is mainly dictated by the accuracy of the turbulence model. Experience from steady flows using algebraic turbulence models has shown that such modelling of turbulence does not provide satisfactory results in most cases. Linear low-Re two-equation models (Launder and Sharma, 1974; Nagano and Kim, 1988) seem to offer the best balance between accuracy and computational cost, but are not able to capture effects arising from normal-stress anisotropy and are less able to predict separation in adverse pressure gradient and shock/boundary-layer interaction (Liou and Shih, 1996; Marvin and Huang, 1996).

At present non-linear models seem to be one of the principal routes for advanced modelling of turbulence beyond the linear eddy-viscosity models. Such models take into account streamline curvature and swirl, as well as history effects. Non-linear models are still being refined and validated for steady flows, mainly two-dimensional and incompressible, (Craft *et al.*, 1996), while more recently experience has been acquired from applications to compressible flows with shock/boundary-layer interaction (Loyau *et al.*1999; Barakos & Drikakis, 1997a).

In the present work, various turbulence closures including algebraic, one-equation as well as linear and non-linear low-Re two-equation models, are validated in transonic buffet flows. The assessment of the models is performed against experimental results (McDevitt and Okuno, 1985) for buffet around the NACA 0012 aerofoil at Reynolds number of 10^7, a range of Mach numbers between 0.7 and 0.85, and for incidence angles between 0 and 5 degrees.

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NUMERICAL METHOD

The numerical simulations have been carried out using an implicit CFD code (Barakos and Drikakis, 1998, 1999) developed for unsteady and turbulent aerodynamic flows. The main feature of the method is the strong coupling of the turbulence closures with the Navier-Stokes equations, via an implicit unfactored scheme and a linear Riemann solver up to third-order accurate.

The compressible Navier-Stokes equations in a two-dimensional curvilinear co-ordinate system (ξ, η), in conjunction with the transport equations of the turbulence model, are written in matrix form as:

$$\frac{\partial U}{\partial t} + \frac{\partial E}{\partial \xi} + \frac{\partial G}{\partial \zeta} = \frac{\partial R}{\partial \xi} + \frac{\partial S}{\partial \zeta} + H \tag{1}$$

U is the six-component vector of the conservative variables:

$$U = J\left(\rho, \rho u, \rho w, e, \rho k, \rho\tilde{\epsilon}\right)^T \tag{2}$$

where ρ is the density, u, w are the velocity components in the x and z directions, respectively, e is the total energy per unit volume, k is the turbulent kinetic energy and $\tilde{\epsilon}$ the isotropic part of the turbulent dissipation rate.

The matrix $H = J\tilde{H}$ has non-zero entries for the source terms of the turbulence model equations. J is the Jacobian of the transformation from Cartesian to curvilinear co-ordinate system. E,G and R,S are the inviscid and viscous fluxes, respectively. The total energy per unit volume e is given as $e = \rho i + \frac{1}{2}\rho(u^2 + w^2) + \rho k$, where i is the specific internal energy. The pressure is calculated by the ideal gas equation of state.

A third-order upwind scheme along with a characteristics-based flux averaging (one-dimensional Riemann solver) is used to calculate the inviscid fluxes at the cell faces (Drikakis and Durst, 1994, Eberle *et al.*, 1992). Limiters based on the squares of pressure derivatives have been used for detecting shocks and contact discontinuities. An implicit-unfactored solver (Barakos and Drikakis, 1998) has been employed for the solution of the equations. A sequence of approximations q^ν such that: $\lim_{\nu>1} q^\nu \to U^{n+1}$. is defined between two time steps n and $n+1$. Using implicit time discretization and after linearizing the fluxes around the sub-iteration state ν the following form is derived:

$$\frac{\Delta q}{\Delta t} + (A^\nu_{inv}\Delta q)_\xi + (C^\nu_{inv}\Delta q)_\zeta - (A^\nu_{vis}\Delta q)_\xi - (C^\nu_{vis}\Delta q)_\zeta = RHS \tag{3}$$

where

$$RHS = -\left(\frac{q^\nu - U^n}{\Delta t} + E^\nu_\xi + G^\nu_\zeta - R^\nu_\xi - S^\nu_\zeta - H^\nu\right) \tag{4}$$

$$\Delta q = q^{\nu+1} - q^\nu \tag{5}$$

and

$$A_{inv} = \frac{\partial E}{\partial U},\ C_{inv} = \frac{\partial G}{\partial U},\ A_{vis} = \frac{\partial R}{\partial U},\ C_{vis} = \frac{\partial S}{\partial U} \tag{6}$$

At each time step the final system of algebraic equations is solved by a point Gauss-Seidel relaxation scheme. According to the present method the transport equations for the turbulence model are solved coupled with the fluid flow equations. This strategy provides fast convergence and compact numerical implementation.

For unsteady flow simulations the first-order in time discretization is replaced by a second-order (Barakos and Drikakis, 1999):

$$\frac{1.5U^{n+1} - 2U^n + 0.5U^{n-1}}{\Delta\tau} = -\left(E^{n+1}_\xi + F^{n+1}_\eta - R^{n+1}_\xi - S^{n+1}_\eta - H^{n+1}\right) \tag{7}$$

For unsteady calculations the time marching in all cells of the computational domain must be performed with the same time step. This global time step is defined for a given CFL number by

$$\Delta\tau \le \Delta\tau_{max} = min\left(J\frac{CFL}{\lambda_{max} + 2\frac{\mu c_p}{Pr}\sqrt{(\xi_x^2 + \xi_z^2 + \zeta_x^2 + \zeta_z^2)}}\right)_{i,k} \tag{8}$$

where λ_{max} is the maximum eigenvalue calculated using the solution from the previous time step.

TURBULENCE MODELLING

In the present study the following models have been employed: the algebraic Baldwin-Lomax (1978) model, the one-equation model of Spalart and Allmaras (1992), the Launder-Sharma (1974) and Nagano-Kim (1988) linear $k-\epsilon$ models, as well as the non-linear eddy-viscosity model of Craft *et al.*(1996) and the non-linear $k-\omega$ model of Sofialidis and Prinos (1997).

In the case of linear eddy-viscosity models the stress tensor t_{ij} is modeled using the Boussinesq approximation:

$$t_{ij} = \bar{t}_{ij} + \tau^R_{ij} \tag{9}$$

where

$$\bar{t}_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij}, \tag{10}$$

$$\tau^R_{ij} = \mu_T\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu_T\frac{\partial u_k}{\partial x_k}\delta_{ij} - \frac{2}{3}\rho k\delta_{ij} \tag{11}$$

and μ_T is the eddy-viscosity.

Non-linear eddy-viscosity models use an expansion of the Reynolds stress components in terms of the mean strain-rate and rotation tensors:

$$S_{ij} = \left(U_{i,j} + U_{j,i}\right)/2, \quad \Omega_{ij} = \left(U_{i,j} - U_{j,i}\right)/2 \tag{12}$$

For the $k-\epsilon$ non-linear EVM of Craft *et al.* (1996) a cubic expansion for the anisotropy of the Reynolds stress tensor, $a_{ij} \equiv \frac{\overline{u_i u_j}}{k} - \frac{2}{3}\delta_{ij}$, has been suggested (Craft *et al.*, 1996):

$$\begin{aligned} a_{ij} = & -\frac{\mu_T}{\rho k}S_{ij} \\ & + c_1\frac{\mu_T}{\rho\tilde{\epsilon}}\left\{S_{ik}S_{kj} - \frac{1}{3}S_{kl}S_{kl}\delta_{ij}\right\} \\ & + c_2\frac{\mu_T}{\rho\tilde{\epsilon}}\left(\Omega_{ik}S_{kj} + \Omega_{jk}S_{ki}\right) \\ & + c_3\frac{\mu_T}{\rho\tilde{\epsilon}}\left(\Omega_{ik}\Omega_{jk} - \frac{1}{3}\Omega_{lk}\Omega_{lk}\delta_{ij}\right) \\ & + c_4\frac{\mu_T k}{\rho\tilde{\epsilon}^2}\left(S_{ki}\Omega_{lj} + S_{kj}\Omega_{li}\right)S_{kl} \\ & + c_5\frac{\mu_T k}{\rho\tilde{\epsilon}^2}\left(\Omega_{il}\Omega_{lm}S_{mj} + S_{il}\Omega_{lm}\Omega_{mj} - \frac{2}{3}S_{lm}\Omega_{mn}\Omega_{nl}\delta_{ij}\right) \\ & + c_6\frac{\mu_T k}{\rho\tilde{\epsilon}^2}S_{ij}S_{kl}S_{kl} + c_7\frac{\mu_T k}{\rho\tilde{\epsilon}^2}S_{ij}\Omega_{kl}\Omega_{kl} \end{aligned} \tag{13}$$

Set	Incidence α (deg.)	Mach Number	Re ($\times 10^{-6}$)
1	2	0.75	1.2–13.9
2	0	0.75	4.0–12.2
3	0	0.8	1.2.0–12.1
4	1	0.8	1.0–10.3
5	2	0.775	1.0–9.9
6	4	0.725	1.0–9.3

Table 1: Nominal conditions for the experiments of McDevitt and Okuno (1985).

This cubic expansion has been utilized here to calculate the components of the Reynolds-stress tensor $-\bar{\rho}\ \overline{u_i u_j}$. In the above, S_{ij} and Ω_{ij} are the strain and vorticity tensors, while $\tilde{S}$ and $\tilde{\Omega}$ are their normalized invariants:

$$\tilde{S} \equiv \frac{k}{\tilde{\epsilon}}\sqrt{S_{ij}S_{ij}/2}, \quad \tilde{\Omega} \equiv \frac{k}{\tilde{\epsilon}}\sqrt{\Omega_{ij}\Omega_{ij}/2} \qquad (14)$$

where the coefficients c_i take the values: $c_1 = -0.1, c_2 = 0.1, c_3 = 0.26, c_4 = -10c_\mu^2$. The eddy viscosity is calculated by: $\mu_T = c_\mu \rho f_\mu \frac{k^2}{\tilde{\epsilon}}$, where

$$c_\mu = \frac{0.3\left[1 - exp\left\{-0.36exp\left(0.75\eta\right)\right\}\right]}{1 + 0.35\eta^{1.5}} \qquad (15)$$

$$f_\mu = 1 - exp\left\{-\left(\frac{\tilde{R}_t}{90}\right)^{1/2} - \left(\frac{\tilde{R}_t}{400}\right)^2\right\} \qquad (16)$$

$$\eta = max\left(\tilde{S}, \tilde{\Omega}\right) \qquad (17)$$

Such functional form of c_μ has been found to be beneficial in flows far from equilibrium and has also been employed in the work by Liou and Shih (1995) for shock/boundary-layer interaction problems. The non-linear $k-\omega$ eddy-viscosity model of Sofialidis and Prinos (1997) is actually the $k-\omega$ version of the non-linear $k-\epsilon$ model of Craft *et al.*(1996).

SIMULATION OF TRANSONIC BUFFET

Test Cases

Computations were carried out for the experimental cases of McDevitt and Okuno (1985). Their experiments have been performed for the NACA 0012 aerofoil at Mach numbers between 0.7 and 0.8, angles of incidence less than 5 degrees and Re_c number between 1 and 14 millions. For the buffet onset they identified the incidence-angle and Mach number as the most important parameters. Their wind-tunnel results are especially suitable for validation of CFD codes since they are free of wall effects in contrast to previous experimental studies (McDevitt and Levy, 1976).

McDevitt and Okuno (1985) organized their findings in six sets of parameters, shown in Table 1. For the sets numbered 4, 5 and 6 buffeting was reported and, consequently, these sets were considered in the present work. As has also been reported by Mateer *et al.*(1992), the effects of boundary-layer tripping on the obtained results is negligible for Re about 10^6. Therefore, computations were carried out for Reynolds number $Re = 10^6$ which provides fully turbulent flow (McDevitt and Okuno, 1985).

Results

Flow unsteadiness around a lifting surface may originate from the surface motion itself or from unsteady boundary conditions. However, there are cases that under certain flow conditions, ie combination of Re and Mach number, can also induce unsteadiness. In the case of transonic buffet, the self-excited shock oscillations are followed by flow separation. It is thus important for a turbulence model to predict accurately the separation induced by the interaction of the shock with the boundary layer and, subsequently, the buffet onset.

Grid	i-direction	k-direction	Far-field location
G1	180	60	5c
G2	241	80	5c
G3	291	85	7c
G4	361	90	7c

Table 2: Details of the computational grids employed in the calculations; Grid G4 was selected for buffet calculations.

In the present study several computational grids have been employed to ensure grid-independent solutions and their details are given in Table 2. In addition, calculations have been undertaken for various dimensions of the computational domain to ensure independence of the solution from the far-field boundary conditions. For buffet predictions the grid G4 (Table 2) was used.

The C_p distributions for $M = 0.775$, $\alpha = 4^o$ using various closures and different grids are compared with the experimental results in Figure 1. For this Mach number and incidence angle, the flow has been found (McDevitt and Okuno, 1985) to be steady and all turbulence models predicted steady flow, as well. As can be seen, none of the models was able to capture exactly the experimental shock position. The non-linear models were used in conjunction with both a functional c_μ (Eq. 17) and a constant c_μ ($c_\mu = 0.09$) coefficient. When the models were employed with a constant c_μ, were found to give results (Figure 1(c)) similar to the ones obtained by the linear models (Figure 1(b)). The results obtained using functional c_μ, however, were in better agreement with the experimental data. Computations without the non-linear expansion revealed that the models predictions were mainly dominated by their damping functions and functional c_μ, and it seems that the anisotropic stress expansion does not play an important role in this case. The results obtained using the Spalart-Allmaras model were comparable to those obtained by the non-linear models using functional c_μ. All linear $k-\epsilon$ models employed in this study predicted the shock position shifted downstream and underestimated the length of separation region. The same was also the case for the algebraic Baldwin-Lomax model (Figure 1(b)).

In Figure 2, comparison of numerical and experimental results for the buffet onset is presented. There is a well-defined region of Mach and incidence angle where buffet occurs. Initially, four computations (Figure 2) were performed at conditions below the experimentally reported buffet onset and steady-state solutions were achieved (symbols in Fig. 2 labelled "no SIO (shock-induced oscillation)"). Afterwards, the incidence-angle was slowly increased to obtain unsteadiness and it was found that after the initial peak of the C_l curve (Figure 3) the computations resulted either in periodic loads, thus indicating buffet (symbols in Fig. 2 labelled "SIO"), or in steady-state flow. In the latter case, the computations were repeated for a higher incidence-angle until buffet is captured. Once buffet was predicted, the incidence-angle was again decreased and the computation was repeated to check whether the experimental boundary (solid line in Figure 2) for buffet onset could be closer approached.

For all combinations of Mach number and incidence an-

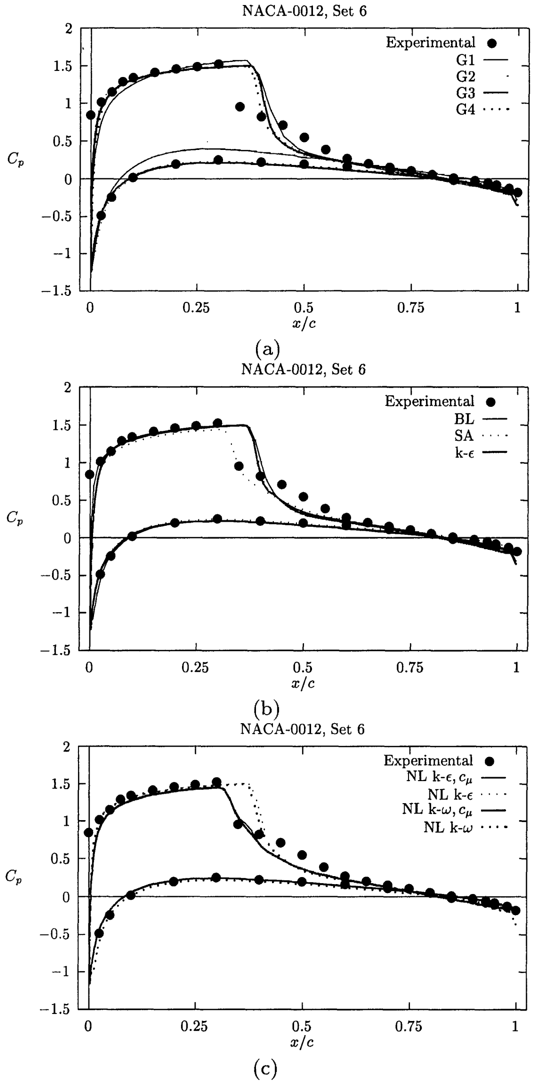

Figure 1: Pressure coefficient distribution around the NACA 0012 aerofoil: (a) grid size effects, (b) comparison between linear turbulence models (c) comparison between non-linear turbulence models; experimental data from McDevitt and Okuno (1985) ($Re = 10^7$, $M = 0.775$, $\alpha = 4^0$).

gle considered here, the linear $k-\epsilon$ models led to a steady solution (Figure 3), thus, failing to predict buffet. As can be seen in Figure 2, the computations predict the buffet onset boundary slightly shifted to higher incidence angles and Mach number. This is in agreement with the calculations of Girondroux and LeBalleur (1988). Edwards (1996), however, reported results closer to the experimental data using an inverse boundary-layer method and the Baldwin-Lomax model. He also found that buffet occurs at $\alpha = 0°$ and Mach number close to 0.83.

The Mach number field is shown in Figure 4 at various instances during the buffet and it is clear that the shock formed on the suction side of the profile changes position in time. A much weaker shock is predicted on the pressure side. The separation region close to the trailing edge of the profile is shown in Figure 5. Initially, the separation region

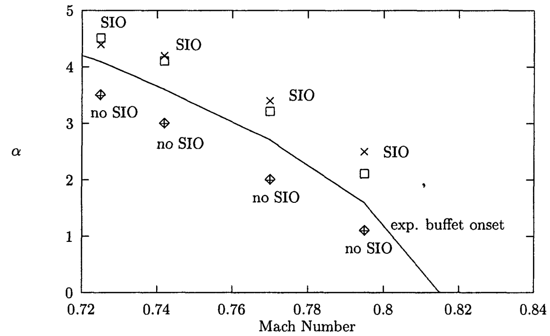

Figure 2: Buffet onset for the NACA 0012 aerofoil ($Re_c = 10^7$, $M = 0.775$, $\alpha = 4.0°$). Solution obtained using the SA model (crosses) and the non-linear $k-\omega$ model (squares); experimental data from McDevitt and Okuno (1985).

increases and extends both towards the trailing edge and the leading edge of the profile. As the first bubble sheds, a second tiny one is formed after the shock (Figure 5(c)) and starts growing again to repeat the cycle.

CONCLUSIONS

Validation and assessment of various turbulence closures has been attempted in transonic flows around aerofoils with buffet. The results revealed that a functional c_μ coefficient significantly influences the models performance. The non-linear stress expansion does not seem to improve the predictions. The effects of c_μ was tested in conjunction with the non-linear models because these models have been calibrated for a functional c_μ. It would be worthwhile to use a functional c_μ in conjunction with a linear $k-\epsilon$ or $k-\omega$ model, but this certainly requires to calibrate the models coefficients in simpler test cases. Furthermore, the results obtained by the Spalart-Allmaras one equation model were found to be comparable to those obtained by the non-linear models with a functional c_μ.

The buffet computations were found to be more computationally demanding than dynamic-stall computations (Barakos & Drikakis, 1997b, 1999), due to the high resolution in time required to resolve the flow unsteadiness. In addition, to predict the buffet onset several computations need to be performed at slightly different conditions and compare the predicted loads.

Future research will try to address both numerical and modelling issues which may affect buffet predictions in transonic flows.

Acknowledgements : The financial support by EPSRC and MoD (GR/L18457) is gratefully acknowledged.

REFERENCES

Baldwin, B.S. and Lomax, H.,1978, "Thin Layer Approximation and Algebraic Model for Separated Turbulent Flows", *AIAA Paper 78-257*.

Barakos, G. and Drikakis, D., 1997a, "Validation of Linear and Non-Linear Low-Re Turbulence Models in Shock/Boundary Layer Interaction", *Eleventh Symposium on Turbulent Shear Flows, TSF-11*, pp. 32-19–32-24.

Barakos, G. and Drikakis, D., 1997b, "Simulation of unsteady aerodynamic flows using low-Re wall-distance-free turbulence models", *ASME Paper FEDSM97-3651*, Fluids Engineering Division, Summer Meeting, Vancouver, Canada, June 22-26.

Barakos, G. and Drikakis, D., 1998, "Implicit-Unfactored

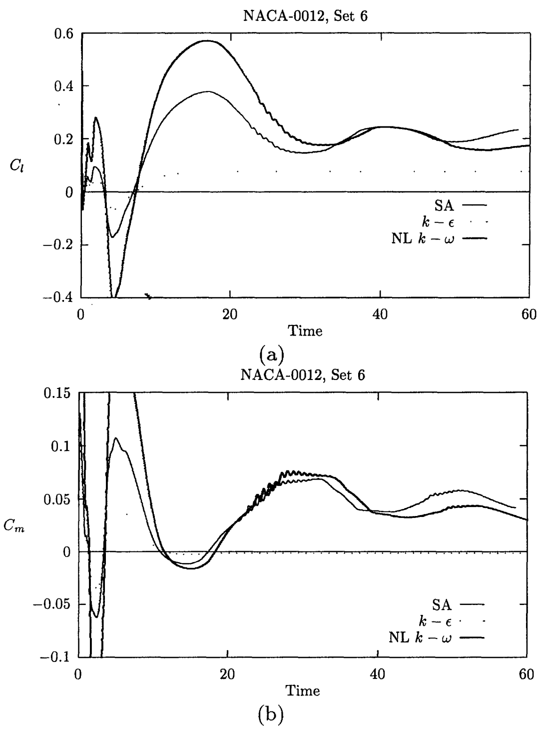

Figure 3: Oscillating airloads for the NACA-0012 aerofoil: (a) lift coefficient, (b) moment coefficient ($Re = 10^7$, $M = 0.775$, $\alpha = 4^0$).

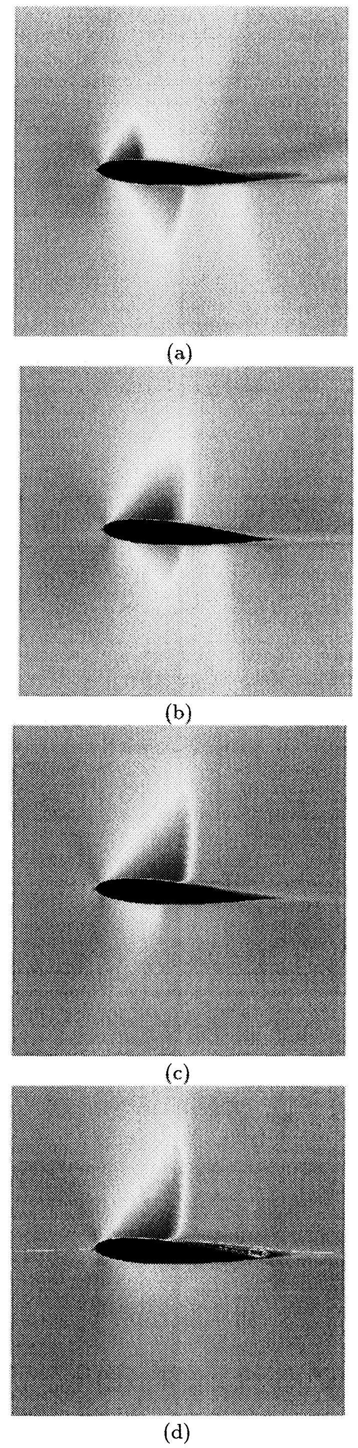

Figure 4: Mach number field around a NACA-0012 aerofoil at different time instants during the buffet development.

Implementation of Two-Equation Turbulence Models in Compressible Navier-Stokes Methods", *Int. J. for Num. Meth. in Fluids*, **28**(1), pp. 73–94.

Barakos, G. and Drikakis, D., 1999, "An Implicit Unfactored Method for Unsteady Turbulent Compressible Flows with Moving Solid Boundaries", *Computers and Fluids*, in press.

Barakos, G., Drikakis, D. and Leschziner, M.A., 1998, "Numerical investigation of the dynamic stall phenomenon using non-linear eddy-viscosity models", *AIAA Paper 98-2740, 16th Applied Aerodynamics Conference*, June 15-18, 1988, Albuquerque, New Mexico.

Craft, T.J., Launder, B.E. and Suga, K., 1996, "Development and Application of a Cubic Eddy-Viscosity Model of Turbulence", *Int. J. Heat and Fluid Flow*, Vol. 17, pp. 108–115.

Drikakis, D. and Durst, F., 1994, "Investigation of Flux Formulae in Transonic Shock Wave/Turbulent Boundary Layer Interaction", *Int. J. Num. Meth. Fluids*, Vol. 18, pp. 385–413.

Eberle, A., Rizzi, A. and Hirschel, E.H., 1992, "Numerical Solutions of the Euler Equations for Steady Flow Problems", Springer Verlag, Wiesbaden.

Edwards, J.W., 1996. "Transonic Shock Oscillations and Wing Flutter Calculated with an Interactive Boundary Layer Coupling Method", *Proc. EUROMECH-Colloquium 349*, Simulation of Fluid-Structure Interaction in Aeronautics, Göttingen, Germany.

Girondroux-Lavigne, P. and LeBalleur J.C., 1988, "Time Consistent Computation of Transonic Buffet Over Airfoils", *ONERA TP No. 1988-97*.

Haase, W., Brandsma, F., Elsholz, E., Leschziner, M. and Schwambron, D. (Eds.),1993, "Euroval, A European initiative on Validation of CFD codes", Notes on Numerical

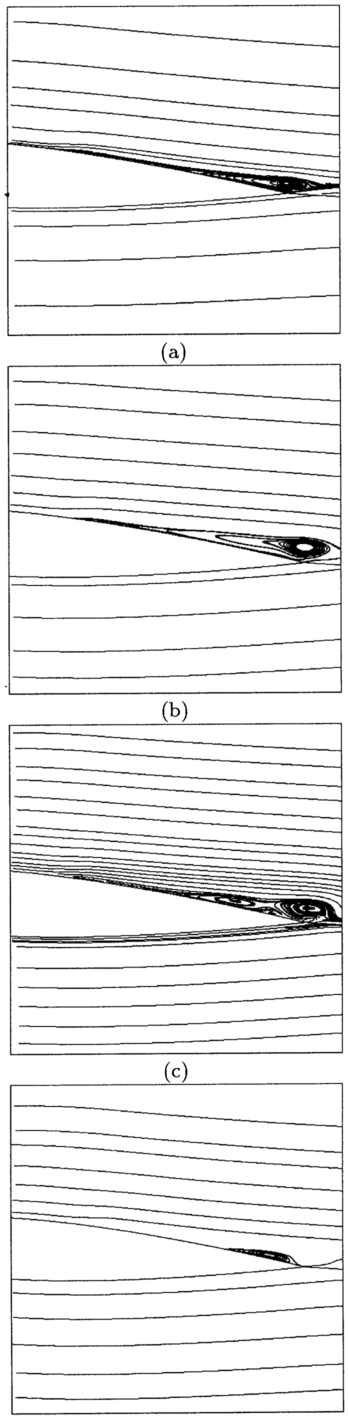

Figure 5: Separated boundary layer around the trailing edge of a NACA-0012 aerofoil at different time instants during the buffet development.

Fluid Mechanics, Vol. 42, Vieweg Verlag.

Launder, B.E. and Sharma, B.I., 1974, "Application of the Energy-Dissipation Model of Turbulence to the Calculation of Flow Near a Spinning Disk", *Letters in Heat and Mass Transfer*, Vol. 1, pp. 131–138.

Loyau, H., Batten, P. and Leschziner, M.A., 1998, "Modelling Shock/Boundary-Layer Interaction with Non-Linear Eddy-Viscosity Closures" UMIST TFD/98/01, 1998.

Leschziner, M.A.,1998, "Experimental needs for CFD validation", ASME Fluids Engineering Conference, Washington DC, June, 1998.

Liou, W.W. and Shih, T.H., 1996, "Transonic Turbulent Flow Predictions with Two-Equation Turbulence Models", NASA CR-198444, ICOMP-96-02, NASA Lewis, OH, USA.

Marvin J.G. and Huang, G.P.,1996, "Turbulence Modeling - Progress and Future Outlook", Keynote Lecture presented at the 15th international Conference on Numerical Methods in Fluid Dynamics, June 1996, Monterey, CA, USA.

Mateer, G.G., Seegmiller, H.L., Hand, L.A. and Szodruch, J., 1992, "An Experimental Investigation of a Supercritical Airfoil at Transonic Speeds", NASA TM-103933, NASA Ames, CA, USA.

McDevitt, J.B. and Okuno, A.F., 1985, 'Static and Dynamic Pressure Measurements on a NACA 0012 Airfoil in the Ames High Reynolds Number Facility', *NASA-TP-2485*, NASA Ames, CA, USA.

McDevitt, J.B., Levy, L.L. Jr. and Deiwert, G.S., 1976, 'Transonic Flow about a Thick Circular-Arc Airfoil', *AIAA J.* Vol. 14, pp. 606–613.

Nagano, Y. and Kim, C., 1988, "A Two-Equation Model for Heat Transport in Wall Turbulent Shear Flows", *J. Heat Transfer*, Vol. 110, pp. 583–589.

Sofialidis,D. and Prinos, P., 1997, "Development of a Non-Linear Strain-Sensitive $k-\omega$ Turbulence Model", *Eleventh Symposium on Turbulent Shear Flows, TSF-11*, Grenoble France, p2-89- p 2-94.

Spalart, P.R, and Allmaras, S.R., 1992, "A One-equation Turbulence Model for Aerodynamic Flows", *AIAA Paper 92-0439*.