TRANSITION TO TURBULENCE IN SPHERICAL GAP FLOWS

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ABSTRACT

The present investigation deals with the transition to turbulence and the behaviour of the turbulent flow in spherical gaps. Different initial and steady or time-dependent boundary conditions result in different ways of the transition from laminar to turbulent flows. The dynamical behaviour is investigated by the power spectra of the local wall shear stress at the outer sphere. The structure of the secondary instabilities is observed by flow visualization. The experimentally observed direct transition to turbulence is analysed analytically using similarity considerations.

INTRODUCTION

Turbulence is important in nature and technology for many reasons. The physical behaviour is investigated by analysing the laminar-turbulent transition and also the fully developed turbulent flows. Surveys are given by Eppler and Fasel (1980), Swinney and Gollub (1981), Barenblatt et.al. (1983), Tatsumi (1984) and Kozlov (1985). In this work the dynamical behaviour of spherical gap flows with and without Taylor vortices is investigated for a small gap width. The dynamical instabilities during the laminar-turbulent transition are analyzed by spectral measurements and visualization of the flow structure. By a quasistatic increase of the Reynolds number the three-dimensional boundary layer flow becomes unstable with respect to time-dependent shear waves, located near the outer sphere. Depending on the basic flow, steady Stuart vortices are also observed near the poles. The superposition of the Stuart vortices and shear waves with increasing Reynolds number leads to a stochastic motion, which occupy at least the whole flow field. A sudden start of the inner sphere to a high supercritical Reynolds number shows a direct onset of turbulence in which the turbulence front spreads from the equator toward the poles in both hemispheres. This paper continues the investigations of Bühler and Zierep (1985) and Bühler and Zierep (1987).

BASIC FLOW IN SPHERICAL GAPS

The experimental set up consist of a rotating inner sphere surrounded by a transparent outer sphere at rest. Silicon oil of a kinematic viscosity $\nu = 5 \cdot 10^{-6}$ m^2/s is used as working fluid. Small aluminium flakes makes the flow structure visible. The wall shear stress is measured with hot film sensors at different meridional positions on the outer sphere. The spherical gap flow depends on the following nondimensional parameters, the Reynolds number and the dimensionless gap width:

$$Re = \frac{R_1^2 \cdot \omega_1}{\nu} \quad , \quad \sigma = \frac{R_2 - R_1}{R_1} = \frac{s}{R_1}$$

The basic flow between two concentric spheres is always three-dimensional. For supercritical Reynolds numbers this flow becomes non-unique. Different modes of flow with and without Taylor vortices exist side by side in the supercritical regime. This fact was first observed by Sawatzki and Zierep (1970) and further studied by Wimmer (1976). Very small gap sizes are investigated by Nakabayashi (1983). A theoretical work on the nonlinear bifurcation was done by Tuckerman (1983) and Marcus and Tuckerman (1987). A comprehensive description of theoretical and experimental investigations is given in Bühler (1985) and Bühler (1990). Investigations for larger gap widths were done by Egbers (1994) and Wulf (1997).

Figure 1 shows the spatial structure of the steady supercritical flow states. They are called Mode I without vortices, Mode III with two Taylor vortices and Mode IV with four Taylor vortices. The structure is plotted with lines of constant streamfunction in the meridional plane.

TIME DEPENDENT INSTABILITIES

These different supercritical flow states are the initial states to observe the laminar-turbulent transition with time-dependent instabilities by increasing the Reynolds number. Figure 2 shows the existence ranges of steady and time-dependent flow. The solid lines represent laminar flows while the dashed lines correspond to turbulent flow. The instabilities are strongly connected to the multiple modes of flow with different local velocity distributions. The numerical solutions (Bühler, 1985) show the boundary layer character of the flows. For higher Reynolds numbers the boundary layers near the inner and outer sphere are separated. It can be shown by analytical investigations, that the thickness of the outer boundary layer is twice of that of the inner one. Figure 3 gives a principle sketch of the flow structure between the two spherical surfaces. The two boundary layers are connected by a nearly rigid body rotation.

MODE I WITHOUT VORTICES

A rapidly acceleration of the inner sphere from rest to a supercritical Reynolds number supress the onset of Taylor vortices near the equator. The existence range for this mode is calculated numerically (Bühler, 1990). The three-dimensional shear waves appear in form of spirals from the poles to the equator for Reynolds numbers Re > 5400. The structure of the shear waves is visualized in Figure 4. The orientation of the waves are perpendicular to the flow near the outer sphere from the equator to the poles. The interaction of these shear waves give rise to a wavy interface at the equator which moves in circumferential direction with the velocity v_w. Figure 3 gives a sketch of the spatial structure of these time dependent shear waves. The structure in the meridional plane is visualized by a light cut method. The streaklines in figure 3 on the right exhibit the structure of the boundary layer instability near the outer sphere. The circumferential velocity v_w of the shear waves depends for a given gap width only on the Reynolds number. The waviness of the equatorial plane has a discrete wave length λ. The dimensionless frequency is therefore direct proportional to the discrete wave number n. The dependence of the frequency on the Reynolds number for different wave numbers are accompanied by hysteresis effects. Characteristic time-dependent signals and the corresponding power spectras of the local wall shear stress are given in Bühler and Zierep (1987). Increasing the Reynolds number further, Stuart vortices are formed near the poles, oriented perpendicular to the shear waves. The interaction between the shear waves and the Stuart vortices leads to a stochastic motion with coherent structures as shown in figure 4 on the right. For Reynolds numbers Re > 8000, the whole flow becomes turbulent. The waviness of the equatorial plane is still concerved for even higher Reynolds numbers.

MODE III WITH TWO VORTICES

This supercritical mode with two vortices near the equator is shown in figure 5. The flow becomes first unstable by Stuart vortices near the poles oriented in the direction of the flow near the outer sphere for Reynolds numbers Re > 6500. For even higher Reynolds numbers Re > 7800 a secondary instability occurs inside the Taylor vortices near the equatorial region. Their frequency is clearly obtained in the power spectra. With increasing Reynolds number shear waves are developed. Their interaction with the Stuart vortices leads to a stochastic motion as can be seen in figure 5 on the right. The Taylor vortices are still conserved within the turbulent motion up to a Reynolds number around 11000 < Re < 14000 , at which the vortices disappear. This is then a transition from mode III into mode I without vortices.

MODE IV WITH FOUR VORTICES

This flow state with four Taylor vortices becomes unstable in the low Reynolds number range 1200 < Re < 2200 with respect to the time-dependent wavy mode. The existence range for the wavy mode is marked by black points in figure 2. After that range a restabilization occurs up to Reynolds numbers Re = 4900. Shear waves are then formed near the poles as can be clearly seen in figure 6. With increasing Reynolds number the interaction between shear waves and Stuart vortices takes place, so that the flow becomes stochastic as can be seen in figure 6 on the right. When the stochastic motion reaches the Taylor vortices, two of them disappear. With a further increase of the Reynolds number the Taylor vortices completely disappear and we have the supercritical turbulent flow without vortices.

DIRECT TRANSITION TO TURBULENCE

A sudden start of the inner sphere from rest to a high supercritical Reynolds number Re > 8000 leads to a direct transition to turbulence. The turbulence first appears at the equator and then the transition fronts in both hemispheres are spreading toward the poles with increasing time. Figure 7 shows the time- dependent process for different times. By the high acceleration of the inner sphere a radial diffusion process takes place. After a characteristic time t_1 a sudden transition from laminar to turbulent motion occurs as it is shown in figure 7. A analytic solution for the position of the onset of the direct transition as function of time are obtained applying the analogy to the Rayleigh-Stokes flat plate problem. The velocity in circumferential direction decreases towards the poles. Therefore the radial diffusion process depends on the meridional coordinate. For the short time limit $t \rightarrow t_1$ we end up with the following analytic expression for the position of the turbulence front:

$$\cos\vartheta^* \cdot \sqrt{\tau} = 1 \quad , \quad \tau = \frac{t}{t_1}$$

The meridional coordinate ϑ^* is measured from the equator and indicates the place of the spreading turbulence front. A comparison between theory and experiment is given in figure 8. The experimental data are for final Reynolds numbers 8000 < Re < 40000. Within the limit of short times there is a good agreement between theory and experiment.

CONCLUSION

The different routes from laminar to turbulent spherical gap flows have been investigated both experimentally and theoretically. The time-dependent shear wave instabilities have an important influence in these transition processes especially in the interaction with the Stuart vortices. The analysis results in an analytic expression for the position of the spreading turbulence front. The combination of experimental and theoretical investigations gives a deeper insight into the process of laminar-turbulent transition. The disappearence of the Taylor vortices at high Reynolds numbers indicates the characteristic difference between cylindrical and spherical Couette flow in the physical behaviour during the laminar turbulent transition and the final turbulent flow.

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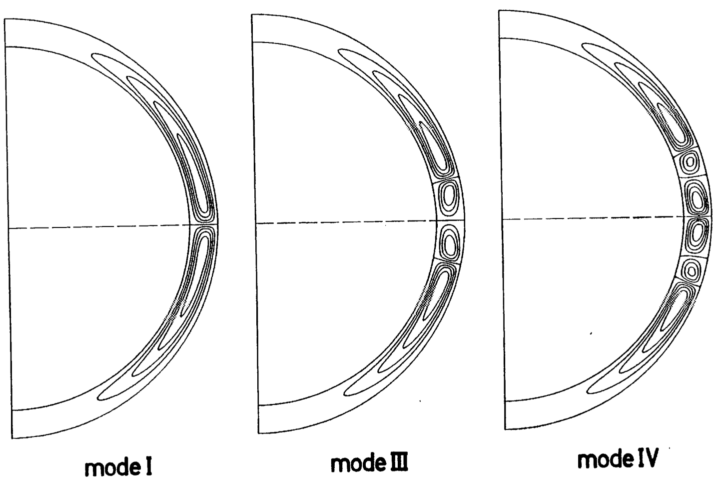

Figure 1. Supercritical non-unique spherical gap flows, different modes at the Reynolds number Re = 2600.

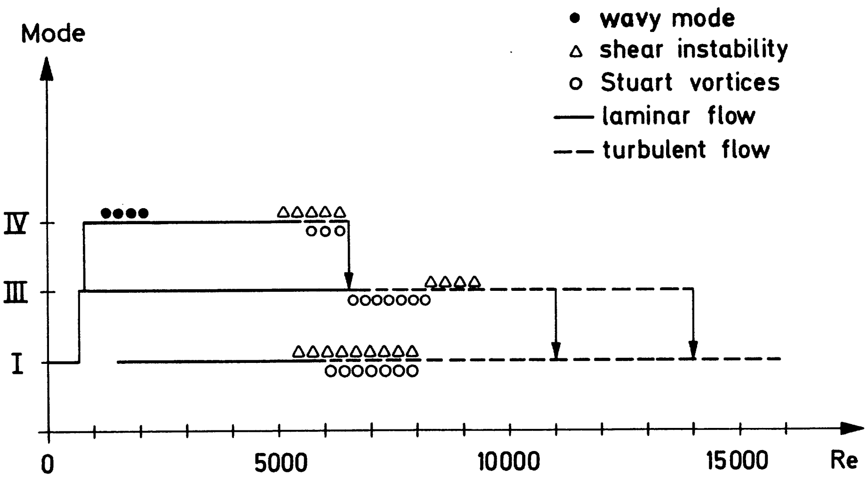

Figure 2. Existence ranges and kind of instabilities as function of Reynolds number, σ = 0,178.

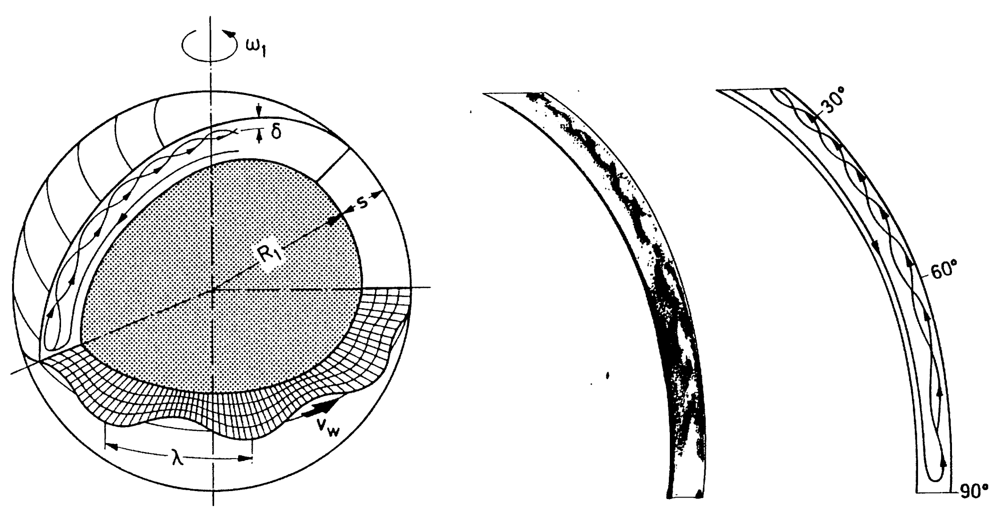

Figure 3. Principle sketch of the spherical gap flow with shear waves (left) and streaklines in the meridional plane (right).

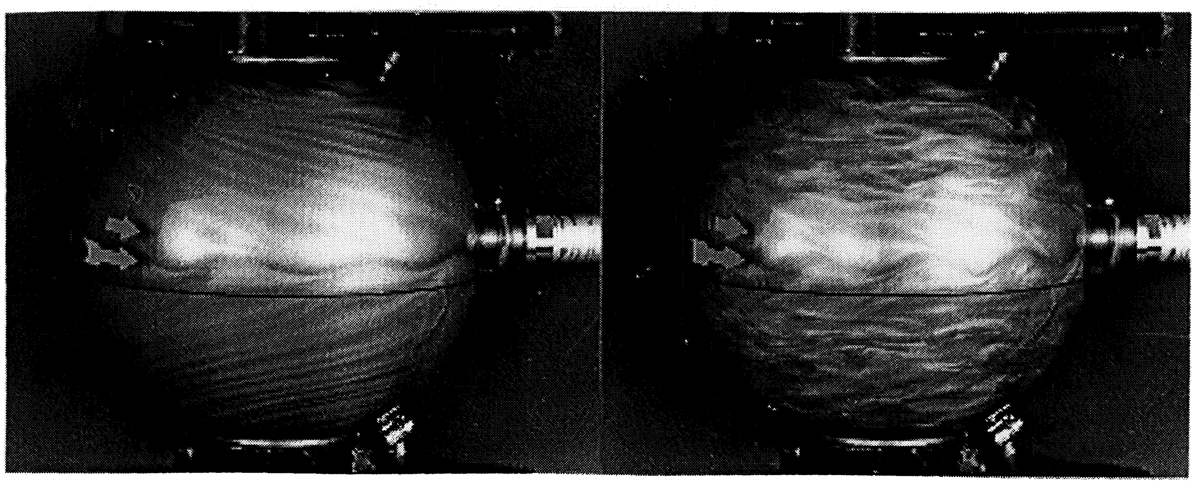

Figure 4. Mode I without Taylor vortices, supercritical basic flow with shear layers Re = 5700 (left) and Re = 6200 (right).

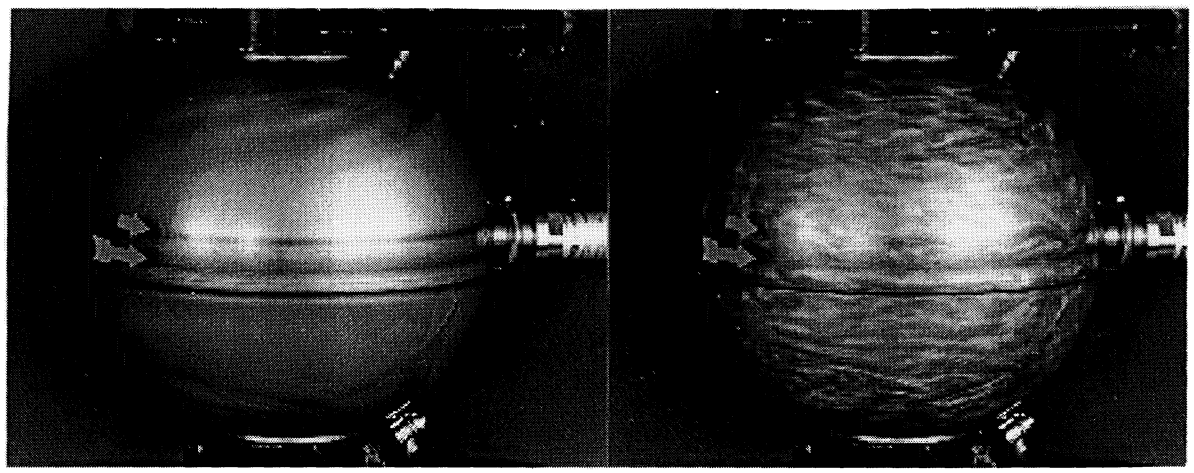

Figure 5. Mode III with two Taylor vortices and Stuart vortices near the poles Re = 7700 (left) and Re = 8500 (right).

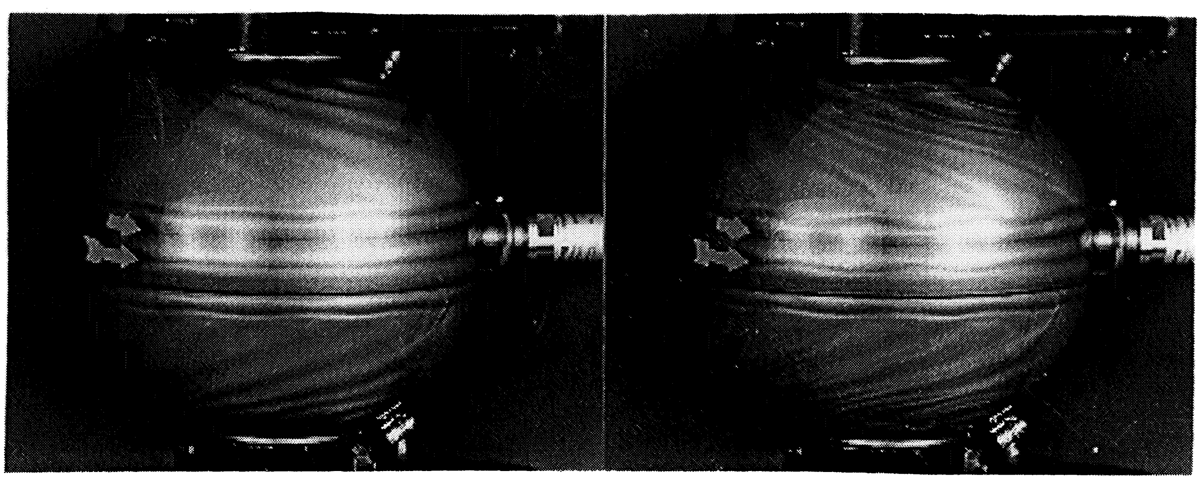

Figure 6. Mode IV with four Taylor vortices and superimposed shear waves Re = 5900 (left) and Re = 6200 (right).

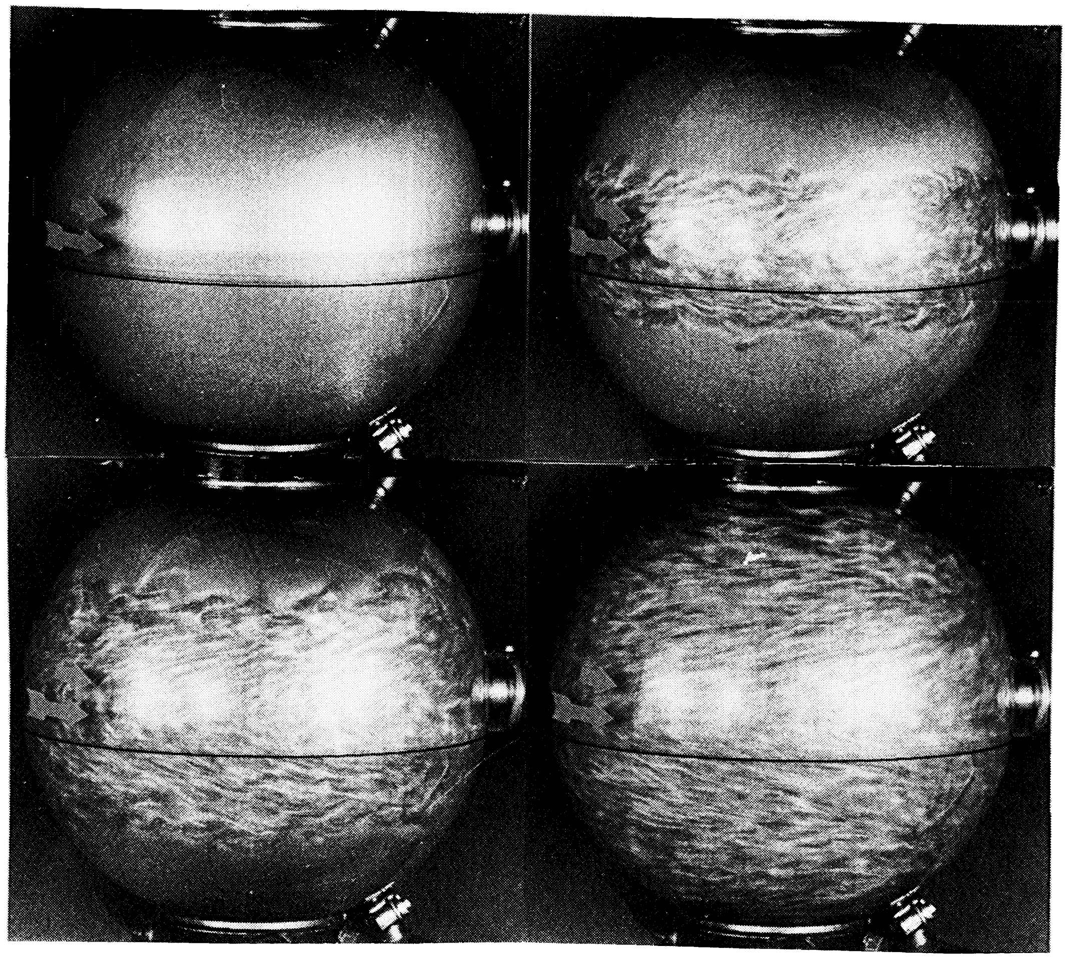

Figure 7. Spreading turbulence front after sudden-start of the inner sphere at different times, Re = 24600.

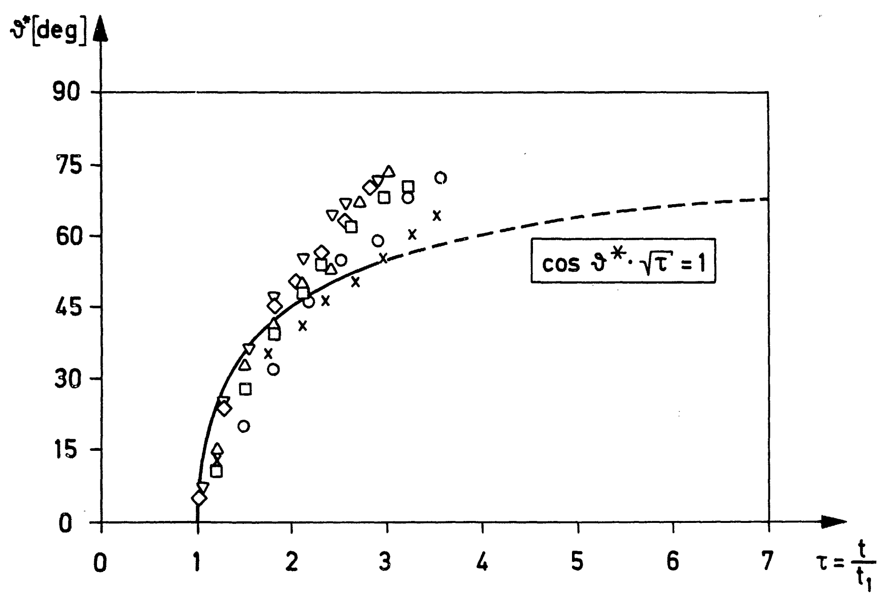

Figure 8. Time-dependent position of the turbulence front, comparison between theory and experiment.