EFFECTS OF RADIUS RATIO AND ASPECT RATIO ON THE SECOND TAYLOR VORTEX FLOW

Q. Xiao, T.T.Lim and Y.T.Chew
Department of Mechanical and Production Engineering
National University of Singapore
10 Kent Ridge Crescent, Singapore 119260

ABSTRACT

Recently, Lim et al (1998) conducted an experimental study on the "history" effect of acceleration on the stability of the Taylor-Couette flow. They found that when the inner cylinder is started from rest, and if the acceleration was higher than a critical value of about 2.2 (s^{-1}), a new flow regime was formed within the Reynolds number range of the wavy vortex flow. In this regime, toroidal vortices in-between the two cylinders showed striking resemblance to the Taylor vortices found at the critical Taylor number. Accordingly, they referred to the new flow as Second Taylor Vortex flow (or STVF). What is fascinating about Lim et al's discovery is that if the acceleration is less than 2.2 (s^{-1}), the vortices are wavy even though the Reynolds number is the same. In this paper, we extended the work of Lim et al, which was restricted to one radius ratio and one aspect ratio only. The primary objectives of this work are to see how changes in the radius ratio and aspect ratio affect the STVF region. The investigation is divided into two parts. In the first part, attention is focused on three different radius ratios (i.e. η = 0.659, 0.8032 and 0.8936) for a fixed aspect ratio Γ of 30. In the second part, the radius ratio is fixed at η = 0.8032 while the aspect ratio is varied systematically from 50 (the value used by Lim et al (1998)) to 20. In both cases, the inner cylinder is subjected to a wide range of acceleration from 0.01(s^{-1}) to 200(s^{-1}). The results, when presented in an acceleration-Reynolds number parametric space, show that STVF flow is not only a function of the Reynolds number and the acceleration as was first reported by Lim et al, but is also a function of the radius ratio, and aspect ratio. Although the exact relationship between then is complex, but in general, with a fixed aspect ratio Γ = 30 STVF regime shrinks with the increase in the radius ratio. As to the effect of the aspect ratio, it is found that for a fixed radius ratio η = 0.8032, STVF regime increases notably with a reduction in the aspect ratio. To the best of our knowledge, these results have not been reported in the literature before. We believe this finding is significant, because it provides an important linkage between the observations of Lim et al and that of Koschmieder for the supercritical Taylor vortex flow.

INTRODUCTION

Since the pioneering work of G. I. Taylor in 1923, Taylor-Couette flow has been the subject of intense theoretical and experimental investigations. One of the most conspicuous characteristics of the Taylor-Couette flow is the non-uniqueness of the wavy flow. This behavior was first reported by Coles in 1965. He found that the flow states depended not only on the initial conditions, but also on the manner in which the inner cylinder was accelerated to the final speed. This finding was subsequently reinforced by another study of Snyder (1969) which found that the axial wavelength of axisymmetric supercritical Taylor vortex flow is very much a function of the initial conditions. Since then, numerous attempts to study the phenomenon have been made by Burkhalter & Koschmieder (1973, 1974), Koschmieder (1979), Park et. al. (1981, 1983), and Andereck et al.(1983).

In a recent investigation by Lim et al (1998) on the history effect of acceleration on the stability of the Taylor-Couette flow, it is found that as long as the acceleration of the inner cylinder (which is defined as a = dRe/dt) is less than a critical value of about 2.2 (s^{-1}), the transition boundaries from one region to another as the Reynolds number increases follows a classical sequence of Circular Couette Flow (CCF) $\Rightarrow$ Taylor Vortex Flow (TVF) $\Rightarrow$ Wavy Vortex Flow (WVF) $\Rightarrow$Turbulent Vortex flow. However, when a > 2.2 (s^{-1}), they discovered a previously unnoticed flow regime in which the vortex pattern shows remarkable resemblance to the Taylor vortex flow. What is interesting about this flow is that it occurs well within the Reynolds number range (3.01 Re_c > Re >1.85 Re_c) of the wavy vortex flow, under quasi-steady condition. Because of its resemblance to the Taylor vortex flow (TVF), they refer to the new flow as a Second Taylor Vortex flow (STVF). When represented in a

Reynolds number-acceleration parametric space, STVF regime is bounded by a parabola-like curve with a minimum occurring at the acceleration of about 2.2 (s^{-1}) (see figure 1 in Lim et al). Just outside the curve, the vortices are wavy with the azimuthal wave number ranging from 2 to 3, depending on the Reynolds number. Although Lim et al have carried out an extensive study of the combined effects of acceleration and the Reynolds number, their findings are somewhat limited because they are restricted to only one radius ratio, aspect ratio (Γ = 50.54, η = 0.8032). An inevitable question arises as to whether the STVF regime is affected by changes in the radius ratio or aspect ratio. Although the study by Burkhalter & Koschmieder (1973, 1974) on the supercritical flow have provided limited answer to the above question, it is far being from conclusive because they are based on two extreme cases of acceleration only (i.e. sudden-start and quasi-steady condition). It is the lack of satisfactory answer that prompted us to carry out the present investigation. The aim of this study is to address the issue of the effect of radius ratio and aspect ratio on the STVF regime under a range of acceleration.

EXPERIMENTAL APPARATUS AND METHODS

The notation used here are following:

H = height of fluid column

R_1 = outer radius of inner cylinder

R_2 = inner radius of outer cylinder

d = gap width (R_2-R_1)

m = azimuthal wavenumber

η = radius ratio (R_1/R_2)

Ω = angular frequency of inner cylinder

Γ = aspect ratio (H/d)

The experimental apparatus consists of an inner rotating cylinder and a stationary outer precision perplex cylinder. The motion of the inner cylinder is controlled by a PC through a micro-stepper motor. To investigate the effect of the radius ratio, three sets of cylinders are used; their detailed geometry is shown in Table 1. To investigate the influence of the aspect ratio, the radius ratio is fixed at η = 0.8032 while the annulus length is varied by altering the height of the working fluid in between the inner and the outer cylinders. The height of the fluid is measured by using a cathetometer. The end conditions of the set-up are such that the annulus is bounded by a stationary solid surface at the bottom, and by a free liquid surface at the top.

The kinematic viscosity (ν) of the mixture is measured by a HAKKE Rheometer at the room temperature of 27°C. All the experimental data are conducted in an air-conditioned room, where the temperature variation is within 0.5°C throughout the experiment. The temperature of the working fluid is monitored continuously and there is no noticeable change during the experiment. To visualize the flow, Kalliroscope AQ-100 reflective flakes are added to the solution, and the resulting motions are monitored and captured with the aids of a 5 Watt Argon ion laser and a CCD video camera.

In this investigation, the two key parameters are Reynolds number (Re) and the acceleration (a). They are defined as Re = $R_1\Omega$ d/ν, and a = dRe/dt = (R_1d/ν) dΩ/dt. Unless otherwise stated, only the inner cylinder is accelerated linearly *from rest* to a desire speed over a predetermined time-interval using the above PC controlled stepper motor. In this paper, the "sudden start" condition is taken to mean the maximum possible average acceleration (dRe/dt $\cong$ 200 (s^{-1})) which could be achieved with the current setup. Likewise, the "quasi-steady" acceleration is defined as the minimum acceleration, below which its effects on the flow parameters under investigation are negligible. For this investigation, it is equal to dRe/dt $\cong$ 0.01 (s^{-1}).

EXPERIMENTAL RESULTS AND DISCUSSIONS

Radius Ratio Effect (Γ = 30, η = 0.6596, 0.8032 and 0.8936)

General Discussion:

In figures 1(a) to 1(c), the effects of radius ratio on the STVF regime and the flow state transition boundary are displayed for the radius ratio of η = 0.656, 0.8032, and 0.8936, respectively. The results are plotted in Reynolds number-acceleration parametric space. Note that, the hatched lines indicate the approximate transition boundary between the wavy vortex flow and the STVF/axisymmetric supercritical Taylor vortex flow. In all the cases, the aspect ratio (Γ = H/d) of the fluid column in-between the two cylinders is 30.

From the results presented, it is evident that STVF regime is not only a function of the acceleration and the Reynolds number as was first reported by Lim et al (1998), but is also a function of the radius ratio. The exact relationship between it and the radius ratio is somewhat complex, but in general, increasing the radius ratio leads to a reduction in the STVF regime and vice versa. In fact, for the largest radius ratio under investigation (i.e.η = 0.8936), the STVF regime is found to decrease considerably (see figure 1(c)) whereas for the smallest radius ratio (i.e. η = 0.6596), the STVF regime has increased to such extent that it resembles the axisymmetric supercritical Taylor vortex flow as reported by Burkhalter & Koschmieder at η = 0.505, Γ = 29.4 and η = 0.727, Γ = 53.38 (see figure 1(a)). For the intermediate radius ratio (i.e. η = 0.8032), the STVF occupies only a small region as depicted in figure 1(b). One significant implication of the discovery of STVF is that for a given Reynolds number in the range of 1.65 < Re/Re_c < 3.15, the flow can assume two distinct flow states, (i.e. either axisymmetric laminar Taylor vortex flow or wavy Taylor vortex flow), depending on the imposed acceleration. This finding is consistent with the results of Lim et al, but is in contrast to the earlier observations made by previous research workers, such as Coles (1965) who reported as many as 20-25 *wavy states* for a given Reynolds number. As far as we are aware, there is no previous report in the literature on the coexistence of two different flow states (which is presented above) at the same Reynolds number, at least, for the case of an inner rotating cylinder with the outer one is stationary.

For the smallest radius ratio (i.e. η = 0.6596), the results are similar to those observed by Burkhalter & Koschmieder (1973, 1974) for the supercritical axisymmetric Taylor vortex flow. In fact, Burkhalter & Koschmieder have found that for the smallest radius ratio (η = 0.505, Γ = 29.4) that they have investigated, the axisymmetric supercritical Taylor flow regime extended from Re/Re_c = 1.0 to Re/Re_c = 9.0. On the other hand, for the largest radius ratio (η = 0.896) used by Koschmieder (1979), the supercritical Taylor vortex flow remains axisymmetric only for a very narrow range of Reynolds number ($1.0 < Re/Re_c < 1.122$). Above this critical Reynolds number (Re/Re_c = 1.122), the flow transforms into a wavy vortex state. Interestingly, Burkhalter & Koschmieder (1973, 1974) did not observe the presence of "parabolic-like" STVF regime as was reported by Lim et al. This implies that the existence of STVF is very sensitive to the radius ratio. Otherwise, Burkhalter & Koschmieder would have observed the STVF regime for the sudden start condition, and a wavy vortex flow for the quasi-steady condition at the Reynolds number range between $1.65 < Re/Re_c < 3.15$.

Based on the discussion presented above, it is reasonable to suggest that the Taylor vortices reported by Lim et al in the STVF regime is the same as the axisymmetric supercritical Taylor vortices observed by Burkhalter & Koschmieder (1973, 1974). Nevertheless, there is one significant difference between the two results. In the study by Lim et al, it is shown that two completely different flow states (i.e. either laminar axisymmetric Taylor vortices or wavy Taylor vortices) can coexist at the same Reynolds number, while the same phenomenon was not observed by Burkhalter & Koschmieder. An important implication of the existence of STVF regime is that it provides a vital linkage between the wavy Taylor vortices and the axisymmetric supercritical Taylor vortices.

Detailed discussion

In this section, we discuss the detailed results of η = 0.8032, follows by η = 0.659 and η = 0.8936. Note that the order of the discussion been chosen so that comparison can first be made with the results of Lim et al conducted at η = 0.8032

η = 0.8032

For this radius ratio, the STVF regime and the flow state transition boundary is displayed in figure 1(b) for a range of acceleration and Reynolds number. Comparison with the study of Lim et al show that the two results are in excellent agreement to within experimental accuracy, although the critical acceleration in the present study appears to be slightly lower. The lower value could be attributed to the difficulty in determining the exact transition boundary. A close examination of figure 1(b) shows that when Re/Re_c = 1.65, the wavy flow changes from an azimuthal wavenumber of m = 2 to m = 3 and the transition is independent of the acceleration.

η = 0.6596

Figure 1(a) shows the flow state transition boundary for the radius ratio of η = 0.659. It is evident that the STVF regime for this radius ratio has increased considerably, extending from as low as the critical Reynolds number (i.e. Re/Re_c =1.0) to as high as Re/Re_c = 8.2. A close examination of the results show that as long as $1.0 < Re/Re_c < 8.2$, the vortex pattern in between the two cylinders is always axisymmetric, and independent of the acceleration. However, beyond Re = $8.2Re_c$, the flow state is very much a function of the acceleration. For example, when Re = $10Re_c$, imposing a high acceleration of $dRe/dt = 200(s^{-1})$ leads to the generation of a wavy vortex flow. On the other hand, a low acceleration of $dRe/dt = 0.01(s^{-1})$ leads to the formation of axisymmetric Taylor vortices, eventhough the final Reynolds numbers for both cases are the same.

η = 0.8936

The acceleration effect on the flow state transition for η = 0.8936 is illustrated in figure 1(c). Here, the small gap size between the two cylinders (i.e. large radius ratio) has led to a considerably decreases of STVF regime. Under this condition, the flow state transition follows a classical sequence of CCF$\Rightarrow$TVF$\Rightarrow$WVF. Interestingly, the transition from CCF to TVF for this particular radius ratio occurs over a very narrow range of $1.00 < Re/Re_c < 1.02$. Beyond Re/Re_c = 1.02, the flow is wavy with the azimuthal wavenumber of m = 1, and as the Reynolds number is increased further to $Re/Re_c < 3.7$, the wave number changes from m = 3 to m = 6. The flow maintains in the same state for a relatively large Reynolds number range before it reverts back to m = 3 when $Re/Re_c \approx 10$.

Regarding the effect of the radius ratio on the onset of wavy vortex flow. It is found that a decrease in the radius ratio delays the onset of wavy vortex flow. For instance, when η= 0.8936, the transitional Re/Re_c = 1.02 whereas for η = 0.659, Re/Re_c changes from 8.2 to 12.0 depending on the imposed acceleration, which is approximately an order of magnitude higher. This finding is consistent with the results of previous research workers.

Aspect Ratio Effects (η = 0.8032, $15 \le \Gamma \le 50.54$)

In figures. 2(a) to 2(d), the effects of aspect ratio on the STVF regime and the flow state transition boundary are displayed for the aspect ratio (Γ) ranging from 15 to 50.54 but η = 0.8032. As before, the results are plotted in a Reynolds number-acceleration parametric space.

$25 \le \Gamma \le 50.54$

In this range of the aspect ratio, it is found that the STVF regime and the flow state transition boundary follows more or less the same trend (see figures. 2(a) and 2(b)), except for the extent of the STVF regime. Here, it can be seen that a reduction in the aspect ratio leads to an increase in STVF region. In fact, within the range of aspect ratio between 40 and 50.54, STVF lies within $1.85 < Re/Re_c < 3.01$ with the critical acceleration (dRe/dt) of $2.2(s^{-1})$. On the other hand, when $\Gamma = 25$, STVF enlarges to $1.35 < Re/Re_c < 3.45$ with $dRe/dt = 0.07(s^{-1})$. It should be pointed out that for these aspect ratios, there exists a narrow region between the critical Reynolds number and the onset of STVF where the wavy vortex flow with an azimuthal wave number (m) of 2 occurs. When the Reynolds number is higher than that indicated by the left-hand transition boundary of the STVF regime, and provided the acceleration is less than the critical value, a wavy vortex flow with m = 3 is generated. This wavy region extends beyond the right-hand transition boundary of the STVF regime as can be seen in figures 2(a) and 2(b)..

$15 \leq \Gamma \leq 20$

In figures 2(c) and 2(d), the flow state transition boundary for the case of $\Gamma = 20$ and 15 are shown. For these aspect ratios it is obvious that the STVF regime has increased to an extent that its left-hand boundary has merged with the Taylor vortex flow and its right hand boundary extends to a higher Reynolds number. These figures clearly show the similarity between the STVF regime and the axisymmetric supercritical Taylor vortex flow as reported by Burkhalter & Koschmieder (1973, 1974). In addition, the critical acceleration which features so prominently in the case of $25 \leq \Gamma \leq 50.54$ seems to be absent here. Also, there is no evidence of the existence of wavy vortex flow down to the acceleration of as low as $0.01(s^{-1})$, which is the lowest acceleration considered in this study. A close examination of figure 2(c) shows that for the aspect ratio of 20, and as long as $1.0 < Re/Re_c < 2.7$, the vortex pattern in-between the two cylinders is always axisymmetric, and independent of the acceleration. However, beyond $Re \approx 2.7Re_c$ (see symbol A in figure 2(c)), the flow state is transformed into wavy vortex flow with m=3, and is slightly acceleration dependent. The result for $\Gamma = 15$ shows a similar trend but with a higher transition point for the onset of wavy flow at $Re \approx 3.61Re_c$. (see also symbol A in figure 2(d)).

From the results presented above, it is found that both the aspect ratio and the radius ratio have a similar effect on STVF. This is perhaps not too surprising because, from geometrical consideration, a reduction in the radius ratio ($\eta = R_1/R_2 = 1-d/R_2$) is equivalent to a reduction in the aspect ratio ($\Gamma = H/d$ with R_2 fixed). Therefore, our two sets of results are consistent with each other.

CONCLUSIONS

Experimental investigations on the effect of radius ratio and aspect ratio have been carried out, and the results show that the STVF regime is not only a function of the acceleration and the Reynolds number as was first reported by Lim et al, but is also a function of the radius ratio and aspect ratio. The exact relationship between them is somewhat complex, but in the acceleration-Reynolds number parametric space, the STVF regime generally decreases with the increase in the radius ratio, and the aspect ratio.

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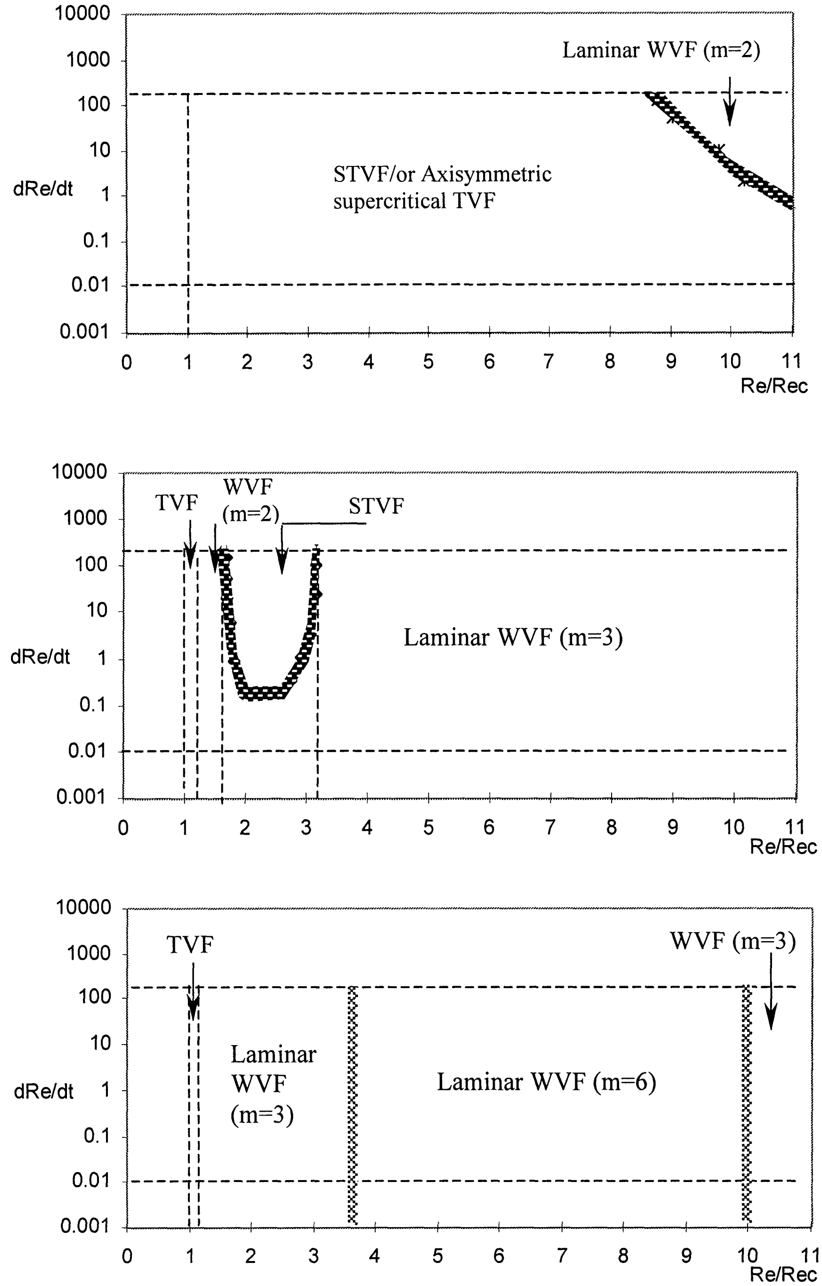
Inner cylinder radius R_1	62.0 ± 0.01 mm	75.5 ± 0.01 mm	84.0 ± 0.01 mm
Outer cylinder radius R_2	94.0 ± 0.01 mm	94.0 ± 0.01 mm	94.0 ± 0.01 mm
Radius ratio $\eta = R_1/R_2$	0.6596	0.8032	0.8936
Fluid Height H	Max: 940mm	Max: 940mm	Max: 940mm
Aspect ratio $\Gamma = H/(R_2 - R_1)$	Max: 30	Max: 50.54	Max: 94
Fluid kinematic viscosity	7.078×10^{-6}	10.50×10^{-6}	11.60×10^{-6}
Horizontal fluid surface	free/upper rigid/bottom	free/upper rigid/bottom	free/upper rigid/bottom

Table 1 Dimensions of the three sets of cylinders used in the investigation



(a) $\eta = 0.6596$
onset of WVF at
Re/Rec = 8.2

(b) $\eta = 0.8032$
(dRe/dt)c = 0.18(s^{-1})
STVF exists 1.65 <
Re/Rec < 3.15

(c) $\eta = 0.8936$
onset of WVF at
Re/Rec = 1.12

Figure 1. Effects of radius ratio on the STVF regime for the aspect ratio $\Gamma = 30$. Note that the hatched lines indicate the approximate transition boundary between the wavy vortex flow and the STVF/axisymmetric supercritical TVF. (a) $\eta = 0.6596$, onset of WVF at Re/Rec = 8.2. (b) $\eta = 0.8032$, (dRe/dt)c = 0.18(s^{-1}), STVF exists 1.65 < Re/Rec <3.15. (c) $\eta = 0.8936$ onset of WVF at Re/Rec = 1.12.

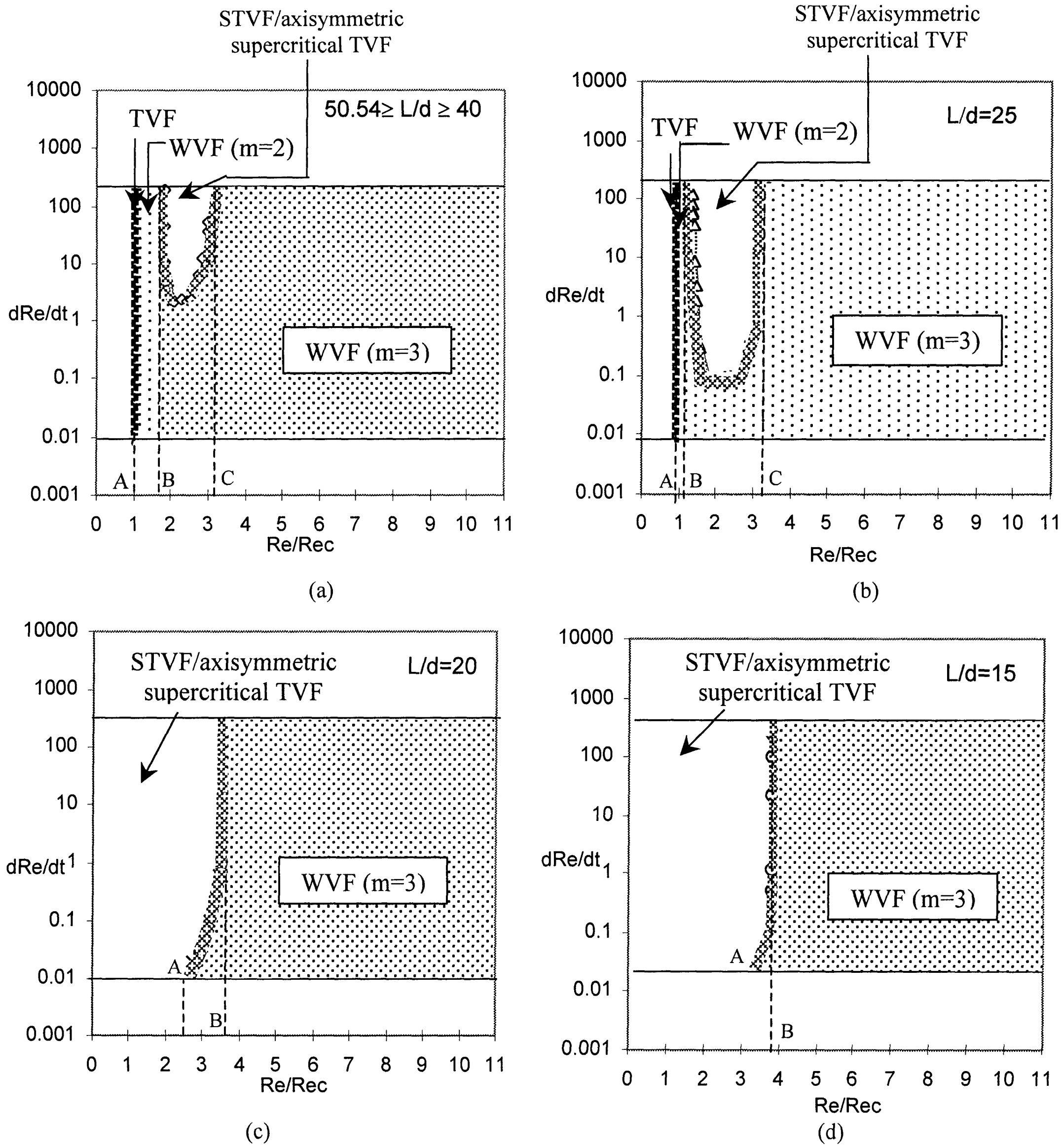


Figure 2 Effects of aspect ratio on the STVF regime. Note that the hatched lines indicate the approximate transition boundary between the wavy vortex flow and the STVF/axisymmetric supercritical TVF. (a) $40 \leq \Gamma \leq 50.54$. Symbol A: onset of wavy vortex flow, $Re/Re_c \approx 1.2$; B: left-hand transition boundary of STVF regime, $Re/Re_c \approx 1.85$. C: right-hand transition boundary of STVF regime, $Re/Re_c \approx 3.01$. (b) $\Gamma = 25$. Symbol A: onset of wavy vortex flow, $Re/Re_c \approx 1.26$; B: left-hand transition boundary of STVF regime, $Re/Re_c \approx 1.35$. C: right-hand transition boundary of STVF regime, $Re/Re_c \approx 3.45$. (c) $\Gamma = 20$. Symbol A: onset of wavy vortex flow for $dRe/dt = 0.01(s^{-1})$, $Re/Re_c \approx 2.7$; B: onset of wavy vortex flow for $dRe/dt = 200(s^{-1})$, $Re/Re_c \approx 3.57$. (d) $\Gamma = 15$. Symbol A: onset of wavy vortex flow for $dRe/dt = 0.01(s^{-1})$, $Re/Re_c \approx 3.61$; B: onset of wavy vortex flow for $dRe/dt = 200(s^{-1})$, $Re/Re_c \approx 3.8$.