MEASUREMENT AND MODELING OF THE TWO-POINT CORRELATION TENSOR IN A PLANE WAKE FOR BROADBAND NOISE PREDICTION

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ABSTRACT

Measurements of the 4-dimensional two-point correlation tensor of a fully developed airfoil wake are presented. These data allow examination of the 'characteristic eddies' of the turbulence through proper orthogonal decomposition and linear stochastic estimation. A simple generic technique has been developed to extrapolate the correlation tensor function from the Reynolds stress field based upon the hypothesis that its form is largely determined by the constraints imposed by inhomogeneity and continuity. Estimates for the plane wake compare favorably with measurements and, interestingly, proper orthogonal modes and characteristic eddy structures inferred from the estimates are similar to those obtained from measurements. This raises some interesting issues.

INTRODUCTION

The motivation behind the present work is the desire for more realistic representations of upwash wave number frequency spectra suitable for use in computing broadband noise resulting from blade-wake interactions in helicopter rotors, stator-wake interactions in aircraft engines, and similar situations. Such calculations (e.g. Amiet, 1975, Glegg *et al.*, 1997, Glegg, 1998) require the two-point space time correlation of the upwash velocity component seen by the lifting surface. Since this information is not generally available the turbulence is usually assumed to behave as though it were homogeneous and isotropic and von Karman's interpolation formula or a similar function is used. In a series of measurements (see Devenport *et al.*, 1998, Devenport *et al.,* 1997, Wittmer *et al.*, 1997, Wenger *et al.*, 1998) we have established that the upwash correlation function in several wake flows is actually quite different than that implied by von Karman's interpolation. The form of the correlation function is dominated by the anisotropy and inhomogeneity visible in the mean flow, and its expression in the large eddy structure of the turbulence. Devenport *et al.* (1998) show that these differences can have a significant effect on the broadband noise generated.

To provide a more realistic representation of the correlation function we have measured the complete two-point space-time correlation tensor for a fully developed airfoil wake. These data have been reduced to a form that we anticipate may be used directly in aeroacoustic calculations. At the same time we have developed a modeling technique that enables us to estimate the two-point correlation tensor function from the Reynolds stress field - such techniques are needed to couple aeroacoustic calculations to RANS calculations.

In this paper we present and compare the measurements and model predictions, concentrating on those features of the instantaneous wake structure that can supposedly be inferred from the two-point correlation tensor. In particular we use the techniques of linear stochastic estimation and proper orthogonal decomposition to extract three-dimensional representations of the 'characteristic eddy structures'. Interestingly, despite the simplicity of our modeling technique, we find that it reproduces many of the gross features of the correlation tensor function. Furthermore, proper orthogonal modes and characteristic eddy structures inferred from the model bear close similarity to those obtained from measurements. This unexpected success implies that the modes may be self-similar and potentially simplifies the aeroacoustic problem.

MEASUREMENTS

Measurements were made in the Virginia Tech Low Speed Wind Tunnel. The empty test section of this tunnel, 2' high and 3' wide, produces a low turbulence (<0.3%), closely uniform flow with near zero streamwise pressure gradient. An 8" chord NACA 0012 airfoil was mounted at the mid height of the test section, at zero sweep and angle of attack. A distributed roughness trip, consisting of a single layer of 0.02"-diameter glass welding beads covering the first 40% of the chordlength of the airfoil, was used to increase the Reynolds number of the airfoil boundary layers and thus its wake.

Single and two-point turbulence measurements were made in the wake of the airfoil using Auspex Corp. Kovaznay-type four-sensor hot-wire probes. These probes are capable of simultaneous three-component measurement from a 0.5mm^3 measurement volume. They were calibrated directly for flow angle using the method of Wittmer *et al.* (1998) - the calibration method having been tested and verified through fully developed turbulent pipe flow measurements. The amplitude and phase response of the separate sensor channels of each probe were optimized, matched and measured by stimulating their impulse response using a pulsed YAG laser. Flat amplitude and matched phase response characteristics were obtained out to well beyond 20KHz - more than sufficient for the present study.

A two-axis computer controlled traverse gear was used to position one of the four-sensor probes in the test section. For the two-point measurements, the second four-sensor probe was held using an unmotorized support. Previous studies by Miranda (1996) suggest that interference between the probes would have been minimal. This conclusion is supported by the present results.

Measurements were made for an approach free stream velocity of 27.5m/s, corresponding to a chord Reynolds number of 328000. Cross section mean velocity and turbulence measurements made 8.33 chordlengths downstream of the foil (x/c=8.33) showed the wake to be closely two-dimensional. Mean velocity, turbulence stress and triple product profiles measured at 18 stations between x/c=0.6 and 11 showed the wake to have a momentum thickness Reynolds number of 3060 and to reach a fully developed state at about x/c=7.5, x/θ=800, with self-similarity in all Reynolds stress and triple product profiles.

Two-point measurements were made at x/c=8.33, x/θ=900. One of the probes (the 'fixed' probe) was placed at 17 positions arranged in a profile from y/L=0 to 3 where y is distance normal to the plane of the wake, measured from its centerline, and L is the half-wake width (defined in terms of its mean-velocity profile). For each of these positions, the second 'moveable' probe was placed at some 400 points each corresponding to a different spanwise (z) and y separation between the probes (up to a maximum of about $4L$). Measurements were made for both positive and negative y-separations, but only for negative z-separations, since the correlation function was expected to be symmetric about z=0. Points were taken at a greater density for smaller separations in anticipation of the shape of the correlation function. The smallest distance between the probes was 0.1", about 16 times the expected Kolmogorov length scale. At each point 50 records of 3072 samples were recorded from each of the 8 sensors at 50kHz - sufficient to calculate a low uncertainty estimate of the cross-spectrum tensor.

The two point cross-spectral estimates were inverse Fourier transformed to obtain estimates of the time-delay correlation for each point pair. These data encapsulate all the empirical information gathered about the two-point correlation tensor but do not form a convenient or useable expression of that function. (The size of the reduced data set is some 500Mbytes.) For this reason the data were interpolated onto a more easily referenced 4-dimensional grid. Care was taken to ensure that the interpolated data contained a faithful representation of the original points. The size of the interpolated correlation tensor function is about 68Mbytes. These data are available as an explicit MATLAB function at http://www.aoe.vt.edu/flowdata/flowdata.html.

The measured correlation $R_{ij} = R_{ij}(y,y',z',\tau)$ depends on the y location of the two points the z distance between them and the time delay τ. It does not depend on absolute z or time since the average flow is homogeneous in these directions. Note that, while x-wise correlation measurement were not performed, these can be inferred from the time delay correlations using Taylor's hypothesis, since the turbulence level in the wake at this station was fairly small (~2%).

MODELING

The objective of the modeling effort was to develop a simple technique for extrapolating the two-point correlation tensor function from single-point Reynolds stress data. We were inspired by the need for such models in aeroacoustics, by the smooth and relatively simple form revealed by the two-point correlation tensor measurements, and by the hypothesis that much of this form might be a direct result of the inhomogeneity of the turbulence field coupled with the constraints of continuity. Our philosophy was to develop the simplest possible generic method consistent with this hypothesis. We define the two-point correlation tensor as

$$R_{ij}(\vec{r},\vec{r}') \equiv \overline{u_i(\vec{r})u_j(\vec{r}')} \tag{1}$$

where $\vec{r} = x_i = (x, y, z)$ and $\vec{r}' = x'_i = (x', y', z')$ denote the two points and u_i (i=1,2,3) denotes the velocity components in the directions of x, y and z. In incompressible flow R_{ij} must be divergence free with respect to both indices, a property that can be guaranteed (Chandrasekhar, 1950) by writing it as the double curl of another tensor function,

$$R_{ij}(\vec{r},\vec{r}') = \varepsilon_{ikl}\varepsilon_{jmn}\frac{\partial^2 q_{ln}(\vec{r},\vec{r}')}{\partial x'_m \partial x_k} \tag{2}$$

For homogeneous turbulence $q_{ln} = -\frac{1}{2}\delta_{ln}u^2 h(|\vec{r}-\vec{r}'|)$ where u is the r.m.s. velocity, and $h()$ is the first moment, with respect to $|\vec{r}-\vec{r}'|$ of the more familiar longitudinal correlation coefficient function $f()$. Note the simplicity in this case of the expression for q_{ln} compared to that needed to explicitly specify R_{ij}.

The simplest possible form for q_{ln} that can work for inhomogeneous turbulence is one in which u^2 is replaced by a scaling function that makes R_{ij} consistent with the single point Reynolds stress data. The expression,

$$q_{\ln}(\vec{r},\vec{r}') = [\alpha_{lo}(\vec{r})\alpha_{on}(\vec{r}') - \tfrac{1}{2}\delta_{\ln}\alpha_{op}(\vec{r})\alpha_{po}(\vec{r}')]h(|\vec{r}-\vec{r}'|) \qquad (3)$$

where α_{ij} is the square root of the specified Reynolds stress tensor, is one such form. The term in the square brackets becomes equal to the Reynolds stress tensor less half its trace when $\vec{r} = \vec{r}'$. If $h()$ retains the properties of the first moment of a correlation coefficient function (i.e. parabaloidal with unit second derivative and zero value at $\vec{r} = \vec{r}'$) then the double curl operation recovers the Reynolds stress tensor.

To apply this model to the airfoil wake we specified the Reynolds stress field from the measured profiles at x/c=8.33 and chose $h()$ to be the first moment of a von Karman spectrum with the same length scale (0.73L) at all points throughout the wake. This value of the length scale was obtained by examining time delay correlations at the wake centerline.

RESULTS

Correlations

Figure 1 compares measured and modeled correlation maps for zero time delay and zero spanwise separation $R_{ij}(y,y',0,0)$. The plots show all 9 components of the correlation function. The lines $y=y'$ represent the Reynolds stress profiles.

The measured data are fairly symmetric about the bottom left to top right diagonal - an indication that they are closely consistent with the requirement that $R_{ij}(y,y')=R_{ji}(y',y)$. Most of the contours of the normal correlations R_{11}, R_{22} and R_{33} are eye-shaped indicating correlations over greater y distances near the wake centerline than at its edges. Not surprisingly correlations of the vertical velocity component R_{22} extend over the greatest vertical distance, those of the spanwise velocity fluctuations R_{33} over the smallest distance. There are also some more subtle features that may be of significance - lobes of negative correlation at larger separation in R_{33}, 'wings' in the R_{11} correlation near the wake center and asymmetry in the R_{12} correlation associated with the dominant Reynolds shear stress.

Many of the gross features of the correlations are quite well reproduced by the model. The model is by no means perfect, however, and there are also a number of differences most notably in the extent of the R_{22} correlation and in the predicted symmetry of R_{12} (symmetry is not necessarily dictated by equation 3). Perhaps the most significant difference between model and measurement is in the time delay correlation of v fluctuations (not shown here). The measured correlation function is oscillatory at larger time delays as a result of quasi-periodicity of the large eddies. This feature is of course absent from the model, since it includes no such physics.

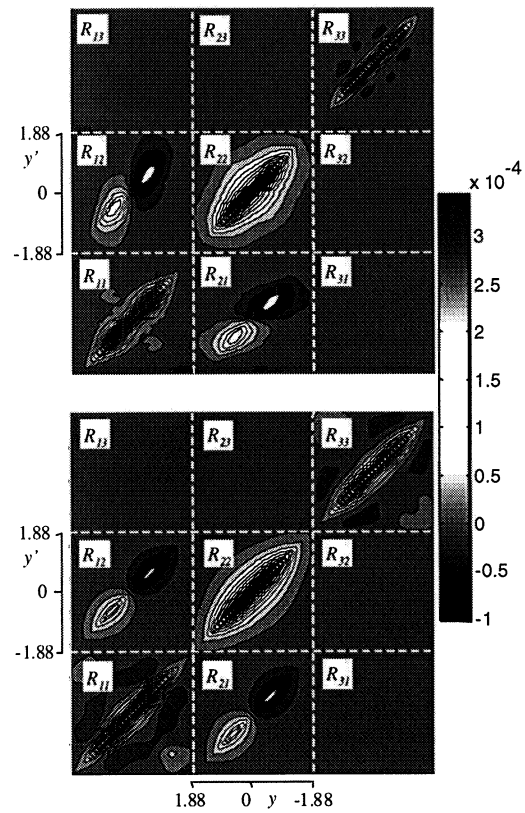

Figure 1. Measured (top) and modeled (bottom) correlation maps for zero time delay and spanwise separation $R_{ij}(y,y',0,0)/U_\infty^2$. Distances in inches, L=0.686"

Characteristic eddy decompositions

It is tempting to use figure 1 to speculate about the form of the instantaneous eddy structures responsible for the measured correlation maps. However, one must remember that this figure shows only one cut through a four-dimensional function and, even at the resolution of the measured data, one could plot some 10000 similar figures for all the possible combinations of z' and τ. One therefore seeks a more objective and efficient method for extracting physical information about the instantaneous flow.

Linear stochastic estimation (LSE) and proper orthogonal decomposition (POD) are both methods for extracting eddy-like velocity fields from correlation functions. LSE (Adrian, 1996) gives the best linear estimate of the instantaneous velocity field given some pre-determined condition, such as the value of a velocity component at one point $u_i(y)$. In this case the estimated velocity field is simply given by the two-point correlation function itself,

$$u_j(y',z',\tau)|_{LSE} = R_{ij}(y,y',z',\tau)\frac{u_i(y)}{\overline{u_i(y)^2}} \quad (4)$$

(no summation implied).

POD (Lumley, 1967) provides a means to compute the optimum basis of the instantaneous velocity field, i.e. a series of orthogonal functions, or modes, that on average provide the best fit to the instantaneous velocity field. If the instantaneous flow can be characterized as a superposition of frequently appearing eddy types, then one might reasonably expect the modes produced by POD to represent the form of those eddies. By maximizing the correlation with the instantaneous velocity field, Lumley shows that the modes are eigenfunctions of the two-point correlation tensor and that their spectrum is given by the corresponding eigenvalues. POD appears to work well in inhomogeneous directions, such as the *y* direction in the present flow. One can for example perform a one dimensional POD of the wake by solving the Fredholm integral (with summation),

$$\int R_{ij}(y,y',0,0)\phi_j(y')dy' = \lambda\phi_i(y) \quad (5)$$

This integral has a multiplicity of eigenfunction solutions $\phi_i^{(n)}(y)$ representing the orthogonal modes of the instantaneous wake velocity profile, and corresponding eigenvalues $\lambda^{(n)}$ (the spectrum) that reveal the proportion of the turbulence kinetic energy produced by each mode.

POD does not work so well in homogeneous directions since here it reduces to Fourier decomposition and sinusoids are obviously not good representations of eddies. The three-dimensional POD of the wake flow involves Fourier transforming with respect to z' and τ (or x) and then solution of the integral,

$$\int R_{ij}(y,y',k_z,k_x)\phi_j(y',k_z,k_x)dy' = \lambda(k_z,k_x)\phi_i(y,k_z,k_x) \quad (6)$$

where k_z is spanwise wavenumber and k_x represents frequency or streamwise wavenumber. Again there are a multiplicity of solutions, but each is multiplied by sinusoidal variations in the homogeneous directions. To construct a three-dimensionally compact representation of an eddy from these modes Berkooz *et al.* (1993) suggest extracting only the dominant mode at each wavenumber combination and then inverse Fourier transforming the result having made assumptions about the relative phasing of these modes. One potential problem with this approach is that it appears to assume a correlation between the dominant modes, when in fact the POD implies the absence of any such correlation.

An alternative approach, which we employ here, is to use POD only to extract the modal profiles in the inhomogeneous direction, according to equation 5. We then use LSE to obtain the best linear estimate of the three-dimensional instantaneous velocity field associated with each complete mode. For the *n*th mode $\phi_i^{(n)}(y)$ we obtain

$$u_j^{(n)}(y',z',\tau)|_{CM} = \frac{1}{\lambda^{(n)}}\int \phi_i^{(n)}(y)R_{ij}(y,y',z',\tau)dy \quad (7)$$

We refer to this approach as the 'combined method'.

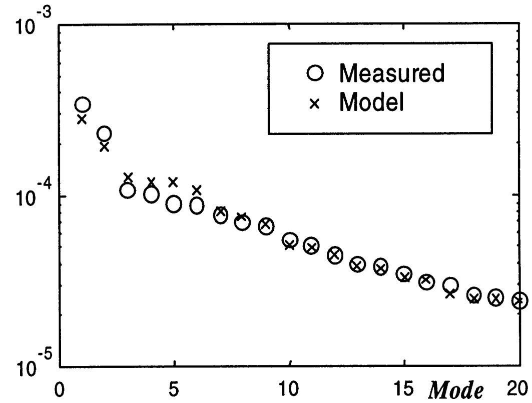

Figure 2. Eigenvalue spectra from the one-dimensional proper orthogonal decomposition in the *y* direction.

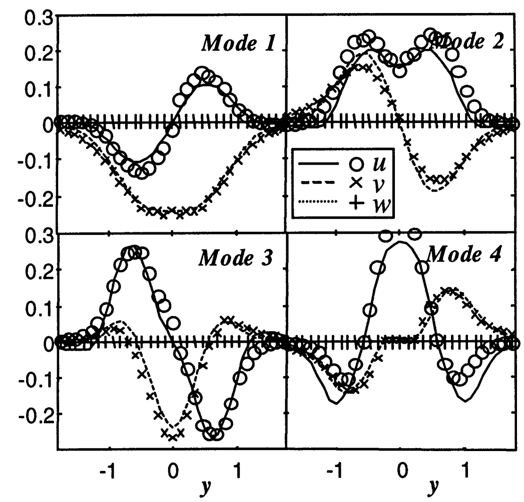

Figure 3. Modal profiles for the first 4 modes. Distance *y* is in inches. Half wake width *L* = 0.686". Symbols show measurements, lines show model results.

Figure 2 shows the eigenvalue spectrum from the solution to equation 5 and figure 3 shows the first 4 modal vector profiles. The measurement resolution allowed for the calculation of about 100 modes, of which the first 40 appeared free of any significant aliasing effects. Figures 4 and 5 show $z=0$ slices through the three-dimensional velocity fields of the characteristic eddy structures obtained using the combined method corresponding to these 4 modes. Results have been computed both from the measured correlation tensor function and from the model function extrapolated from the Reynolds stress profiles.

The measured eigenvalue spectrum (figure 2) shows that the first two modes contain significantly more energy than the others. Together these two modes account for about 27% of the total t.k.e. in the wake. The corresponding modal

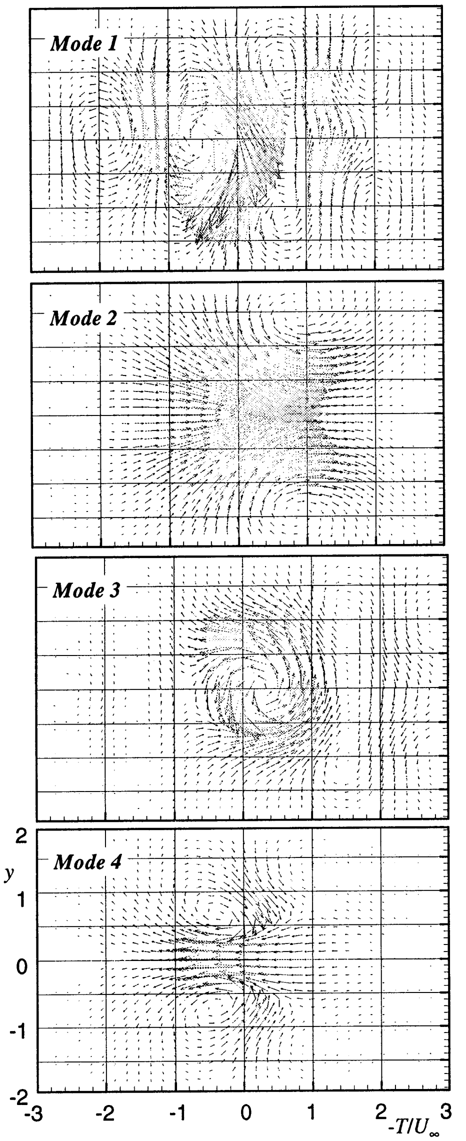

Figure 4. Characteristic eddy structures corresponding to the first 4 modes deduced from the measured two-point correlation tensor. Distances in inches, L=0.686".

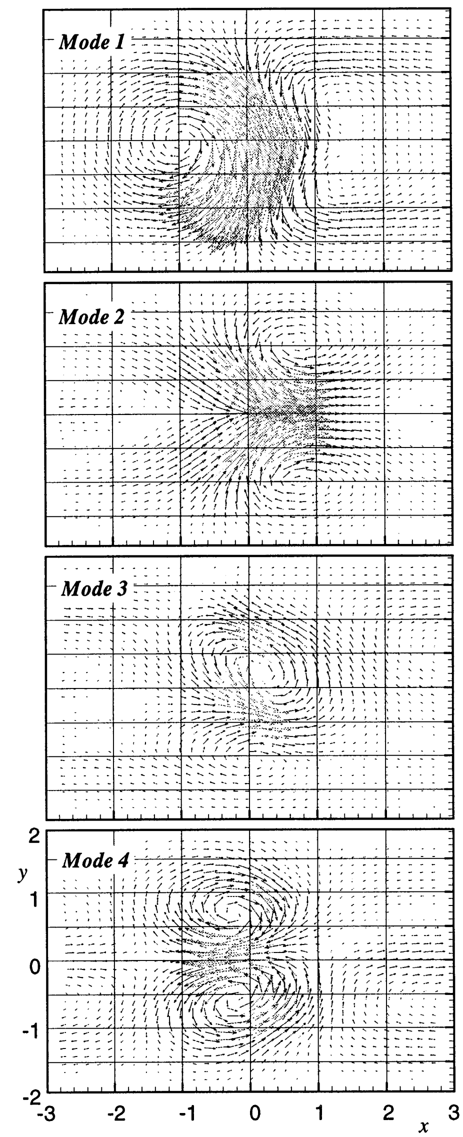

Figure 5. Characteristic eddy structures corresponding to the first 4 modes deduced from the model two-point correlation tensor. Distances in inches, L=0.686".

profiles (figures 3a and 3b) are clearly associated with the generation of the Reynolds shear stress since they combine symmetric and antisymmetric u and v profiles. The third and fourth modes appear to imply more complex motions (w is negligible in the first 4 modes). The corresponding characteristic eddies (figure 4) appear to be predominantly spanwise roller-type structures appearing either singly (modes 1 and 3) or in symmetric pairs (modes 1, 2 and 4). The remainder of these velocity fields away from the z=0 plane reveal no new significant structure. In particular, there is no sign of the so-called double roller eddy seen by Payne and Lumley (1967), among others, in cylinder wakes. The absence of such a structure is not in itself surprising since it is well known that the structure of a fully developed wake is strongly dependent on its initial condition.

The results inferred from the model correlation tensor function should, of course, show none of these features since the model contains no physics beyond the Reynolds stress profile and a single length scale. However, the modal spectrum (figure 2) obtained from the model is very similar to that measured and the model seems to provides an accurate prediction of the modal profiles (figure 3). Furthermore the model reproduces the dominant features of all the associated three-dimensional characteristic eddy structures (figure 5). Estimates of the three-dimensional velocity fields associated with the wake eddies based upon LSE alone (through application of equation 5) show similar agreement between model and measurement.

One could interpret these results as demonstrating that the instantaneous form of the dominant wake eddies can be predicted merely from the Reynolds stress distribution and a length scale. The alternative (and perhaps more realistic) interpretation is that the characteristic eddy structures are in fact not representative of the instantaneous eddies at all, but merely eddy-like representations of the correlation function. Either way, it appears that the modes and associated velocity fields can, at least approximately, be predicted from the Reynolds stress field. This in turn implies that the modes are self-similar. Both observations provide important simplifications to the aeroacoustic problem of calculating the response of a downstream airfoil or structure to a wake flow.

CONCLUSIONS

1. The 4-dimensional two-point correlation tensor function of the fully developed turbulent wake of an airfoil has been measured.
2. A compact representation of this function, suitable for use in aeroacoustic calculations, has been extracted from these measurements.
3. A simple generic technique has been developed to extrapolate this information from the Reynolds stress field. Applied to the wake this reproduces many of the gross features of the correlation function, including the dominant proper orthogonal modes and their spectrum.
4. Characteristic eddy structures inferred from the measured two-point correlation tensor do not appear to contain any more physical information about the instantaneous eddies than that which can be inferred from the Reynolds stress profile and a single length scale.
5. The similarity in characteristic eddy structures inferred from the measured and model correlation functions implies that these modes are self-similar. Further measurements are needed to verify this.

ACKNOWLEDGEMENTS

The authors would like to acknowledge NASA Langley, in particular Casey Burley and Tom Brooks, for their support under grant NAG 1-1942. We would also like to thank Roger Alexander for his assistance in the initial analysis of the two-point measurements.

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