VORTICITY FIELD EVOLUTION IN A FORCED WAKE

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ABSTRACT

Previous studies have shown that forcing a low Reynolds number, confined 2-D wake may lead to greatly enhanced molecular mixing. This study presents whole-field measurements of the velocity and vorticity fields in such a flow in order to examine its underlying flow structure and the features connected to mixing enhancement. The variation of the spanwise vorticity field versus forcing amplitude is presented. Results show that the downstream location where the magnitude of the mean velocity gradient $\partial w/\partial z$ starts to increase coincides closely with the location where mixing has been noted to increase. The downstream evolution of the streamwise vorticity ω_x shows that the high amplitude forcing cases corresponding to significant mixing enhancement are characterized by large values of ω_x. For these cases, the regions of ω_x move quickly away from the facility sidewalls towards the center-span of the test section. In contrast, these regions remain close to the sidewalls in the low amplitude forcing conditions which result in a modest mixing increase.

INTRODUCTION

Forcing a low Reynolds number, confined 2-D wake can lead to a highly three-dimensional flow and a large increase in mixing. Estimates of the amount of molecularly mixed fluid in a liquid-phase flow made by Koochesfahani and Nelson (1997) have shown that the mixed fluid fraction in this forced, low Reynolds number flow is 2.5 to 3 times larger than that seen in high Reynolds number, natural two-stream liquid mixing layers and 60% larger than that in gas-phase turbulent shear layers.

Figure 1 shows the downstream development at midspan of the mixed fluid fraction, δ_p/δ_1, at a fixed frequency for three different forcing amplitudes from Nelson and Koochesfahani (1997). The mixed fluid fraction is defined here as the ratio of product thickness measured by a chemically reacting LIF technique to the local wake width. Note in Figure 1 that forcing at low and moderate amplitudes results in a modest increase in the amount of mixed fluid. The high amplitude forcing case, however, results in a significant increase in δ_p/δ_1. The leveling out of the profile for $x > 20$ cm is caused by the confinement imposed by the top and bottom walls of the flow facility.

Earlier studies by Roberts (1985) show that mixing enhancement is also seen when the forcing frequency is varied. Significant increases in the amount of mixed fluid were found in this study when the flow was forced at twice the natural shedding frequency. The mixing studies of Roberts, Koochesfahani and Nelson, and MacKinnon and Koochesfahani (1997) have all noted that, for cases of large mixing enhancement, the mixing regions start near the side walls of the facility and move toward center-span farther downstream. A simple vortex model was proposed by Roberts to explain this feature. The mixing regions near the side walls were modeled by a pair of counter-rotating streamwise vortices formed as a result of the reorientation of the primary spanwise wake vortices due to the side-wall boundary layer. The induced flow field of this vortex system, and its images, cause the lateral movement of these vortical regions toward the center-span.

All previous studies of the wake flow discussed here have concentrated on either flow visualization or quantification of the mixing characteristics. While all these studies connect the increase in mixing to the downstream evolution of the streamwise vorticity, no information has been available on the behavior of the vorticity field in this flow. In the present study, the vorticity evolution is quantified using Molecular Tagging Velocimetry (MTV), and the structure of the velocity and vorticity fields are explored in the context of previously observed mixing enhancement.

EXPERIMENTAL FACILITY

The experiments were conducted in a gravity driven, liquid two-stream mixing layer facility located in the Turbulent Mixing and Unsteady Aerodynamics Laboratory at Michigan State University. This is the same facility used in the mixing studies conducted by MacKinnon and Koochesfahani (1997), Koochesfahani and Nelson (1997), and Nelson (1996). The "high", "middle", and "low" amplitude forcing in these studies (see also Figure 1) correspond to velocity perturbation amplitudes of 11.6%, 7.3%, and 2.1%. In this facility, fluid pre-mixed with chemicals necessary for the molecular tagging diagnostic technique are pumped into two constant head reservoir tanks approximately 3.5 m above the test section. The operational parameters of the experiments were selected to be the same as previous mixing studies in order to allow direct comparison. The free-stream speeds of the two sides were set to $u_1 = u_2 = 9.4$ cm/s to generate a wake flow with an initial Reynolds number of about 100 based on the wake momentum thickness.

Two-dimensional perturbations were introduced into the upper free-stream of this flow by means of an oscillating bellows mechanism in the upper supply line. The mechanism was driven by an electromagnetic shaker whose command signal originated from a standard function generator. The command signal was recorded for each experiment to facilitate the phase-averaging of the measured data which will be discussed in the next section. Sinusoidal perturbation of the flow at different amplitudes and a fixed frequency of 6 Hz, which is approximately the natural shedding frequency, were examined in this study. It should be noted that even though forcing is only applied to the upper stream, oscillations are observed in the free-stream region in the bottom half of the test section that are nearly in phase with those in upper free stream.

Measurements were made over multiple planes as shown in Figure 2. The streamwise planes yielded the (u, v) velocities in the (x, y) plane as well as the spanwise vorticity, ω_z. The cross-stream planes yielded the (v, w) velocities in the (y, z) plane as well as the streamwise vorticity, ω_x. The coordinate system for these measurements is also shown in this figure. Approximately 600-800 simultaneous velocity measurements are made over each plane. The results described in this paper represent only a small portion of this data set related to the issues addressed herein. The complete details of the development of the velocity and vorticity field of this flow can be found in Cohn (1999).

VELOCITY/VORTICITY MEASUREMENT METHOD

The velocity field was measured using Molecular Tagging Velocimetry (MTV). MTV is a full-field optical diagnostic which allows for the non-intrusive measurement of the velocity field in a flowing medium. This technique can be thought of as the molecular equivalent of Particle Image Velocimetry (PIV). Rather than tracking particles placed in the flowing medium, molecular regions are tracked by recording the luminescence of the tracer molecules. A complete description of the implementation of this technique, and the parameters necessary for an optimal experiment, can be found in Gendrich and Koochesfahani (1996), Gendrich *et al.* (1997), and Koochesfahani *et al.* (1997).

Based on the quality of the images and conditions in the current experiments and the work of Gendrich and Koochesfahani (1996), it is expected that the instantaneous displacement of tagged regions is measured with a 95% confidence interval of less than 0.1 pixel. This corresponds to uncertainties of 0.15 cm/s and 0.2 cm/s for the instantaneous streamwise and cross-stream plane velocity measurements, respectively.

The ω_z and ω_z fields are calculated using a 2nd order central (4th order accurate) finite difference technique from their respective velocity fields. The use of standard uncertainty analysis indicates that the uncertainty in the vorticity field is given by $\delta\omega = 1.34\ \delta u/h$, where h is the grid spacing used in the differentiation and δu is the uncertainty in the velocity measurement. Applying the parameters from the current experiment, the 95% confidence level for the instantaneous vorticity in the current experiments corresponds to 1.6 s^{-1} for ω_z and 1.8 s^{-1} for ω_x.

Previous studies of this forced wake suggest that many features of the flow are quite repeatable and phase-locked to the forcing frequency. This allows a phase-averaging procedure to be used to relate data from different fields of view. Results presented in Cohn (1999) show that the phase-averaged data is a very good representation of the instantaneous result in this flow. Added benefits of phase-averaging are the reduction of the measurement uncertainty and an increase of the "effective" temporal sampling rate of the measurements.

Using the bellows command signal as a reference, the velocity field data are phase ordered and placed into 64 non-overlapping phase bins. For each field of view, 1000 consecutive velocity realizations are acquired at a sampling rate of 33.3 ms. There are approximately 16 instantaneous realizations in each bin, which are then averaged. With 16 realizations per bin, the measurement uncertainty is reduced by a factor of 4 compared to the instantaneous values mentioned earlier. Using 64 bins per forcing cycle, the effective temporal spacing between consecutive phase bins is equivalent to 2.6 ms in real-time.

SIDE VIEW RESULTS AT CENTER SPAN

Figure 3 illustrates the downstream development of the phase-averaged spanwise vorticity field, $\langle\omega_z\rangle$, for four forcing amplitudes as well as an instantaneous map from the unforced case. Note that each forcing case combines data from 5 planes covering different streamwise ranges using the known phase information. The vorticity level is indicated by the grayscale level, and also by contour lines placed every ± 5 s^{-1}.

In the unforced case, the vorticity shed from the splitter plate turns into an organized vortex array at about x = 6 cm. However, the horizontal spacing between the structures is evolving until approximately 10 cm downstream. By x = 17 cm, the magnitude of the vorticity within the spanwise rollers has decreased to a very small magnitude.

When forcing is applied, identifiable vortical structures are apparent at the earliest upstream location. With increasing forcing amplitude, the peak vorticity level of the vortical structure increases in the upstream locations. In all cases the peak vorticity decreases as the flow moves downstream with a rate of decrease which depends upon the forcing amplitude. In fact, at the farthest downstream locations, the peak $<\omega_z>$ of the high amplitude cases are less than that of the low amplitude cases. This is seen more clearly in Figure 4 which shows the effect of downstream location and forcing amplitude on the peak levels of $<\omega_z>$. When interpreting the absolute values of the measured vorticity, it should be noted that the vorticity is underestimated due to the spatial averaging inherent in the measurement. This mean bias error has been estimated in Cohn (1999) to be about 18% for the conditions of these experiments.

The downstream decrease of $<\omega_z>$ in Figure 4b is not primarily the result of viscous diffusion. For the forcing amplitude of 11.6% the estimate of the effect of viscous diffusion is given in Figure 4b. Also given is the estimate of the effect of the vorticity stretching term $[(\partial w/\partial z)\ \omega_z]$ in the vorticity transport equation. A value of $\partial w/\partial z = -1\ s^{-1}$ was used, which is consistent with our measurements of this quantity at center-span. Note that this corresponds to "negative" stretching and a reduction of the vorticity level. It is clear in Figure 4b that the decay of peak vorticity by negative stretching dominates over viscous diffusion and is consistent with the measured decay.

The measured u(x,y), v(x,y) data can be used to estimate the value of $\partial w/\partial z$ based on the continuity equation in order to provide an indication of the extent of three-dimensionality in the flow. The uncertainty in the instantaneous value of $\partial w/\partial z$ is the same as that for the vorticity described earlier. The downstream variation of the mean $\partial w/\partial z$ at center-span is depicted in Figure 5 for the case of 11.6% forcing amplitude. Comparing this result with the earlier mixing data suggests that the increase of three-dimensionality indicated by the increase in the magnitude of $\partial w/\partial z$ is consistent with mixing enhancement. Note that mixed fluid fraction, δ_p/δ_1, begins to increase just after the magnitude of the mean $\partial w/\partial z$ increases.

END VIEW RESULTS

Figure 6 compares the mean streamwise vorticity field for the unforced case as well as for the 7.3% and 11.6% forcing amplitudes at four downstream locations. Only the right half-plane of the test section is shown. In the unforced case, the instantaneous, and thus also the mean ω_x values are very small. For the 7.3% forcing amplitude, a pair of counter-rotating regions of vorticity are seen near the sidewall located at z = -4 cm. Note that these regions remain close to the sidewall as the flow moves downstream, and there is no evidence of the presence of ω_x at center-span. Therefore, the modest molecular mixing enhancement at center-span at this amplitude, see Figure 1, is not due to small scales generated by an increase in three-dimensionality. Rather, it is due to the additional "jelly-roll" mixing interface, as seen in MacKinnon and Koochesfahani (1997), which is caused by the increased spanwise vorticity.

The 11.6% forcing case also initially exhibits a pair of counter-rotating vortices near the sidewall with a much higher vorticity level. However, farther downstream multiple regions of ω_x are apparent and they move towards the center of the test-section. Both of these effects are responsible for the increased three-dimensionality and the observed mixing enhancement at these high amplitudes. We draw attention to the fact that the regions of streamwise vorticity are much more complex than the simple vortex pair model of Roberts (1985). The presence of multiple regions of ω_x has been argued by Cohn (1999) to be the result of the reorientation of the spanwise vortex array convecting by a given spatial location.

CONCLUSIONS

The evolution of the vorticity field in a forced low Reynolds number confined wake has been presented. It is found that the peak vorticity of the spanwise vortices decays with downstream distance primarily due to negative stretching. The increase in the magnitude of mean $\partial w/\partial z$ appears to be a good indicator of mixing enhancement for the case studied.

For a medium forcing amplitude with modest mixing enhancement, there is little evidence of streamwise vorticity in the center-span. The mixing increase in this case is the results of additional mixing interface generated by the spanwise vorticity. In the high amplitude forcing case, the combination of higher streamwise vorticity levels and the inward movement of the regions of ω_x leads to the previously recorded molecular mixing enhancement at center-span.

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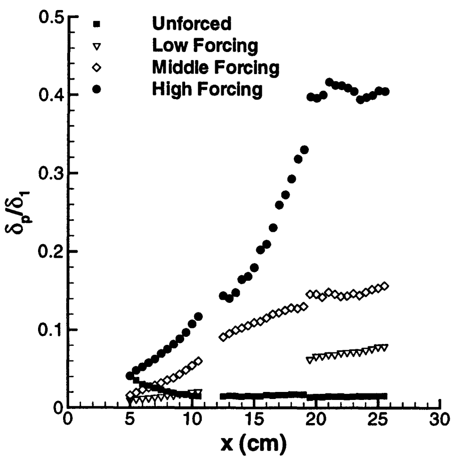
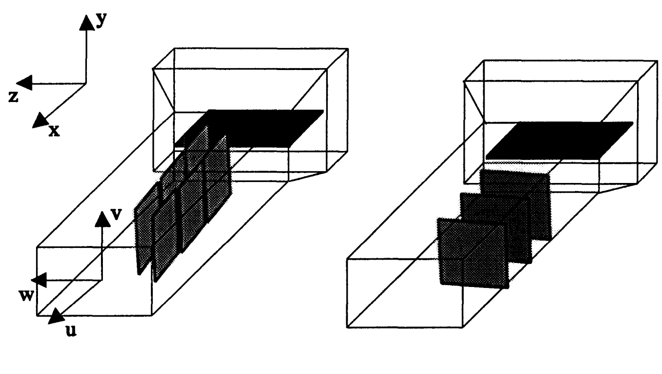


Figure 1: Streamwise variation of mixed fluid fraction at mid-span for different forcing amplitudes (Koochesfahani and Nelson, 1997).



(a) (b)

Figure 2: MTV measurement planes. (a) Streamwise (side view) measurement planes. (b) Spanwise (end view) measurement planes. Note that the origin of the coordinate system is at the center of the test section at the tip of the splitter plate.

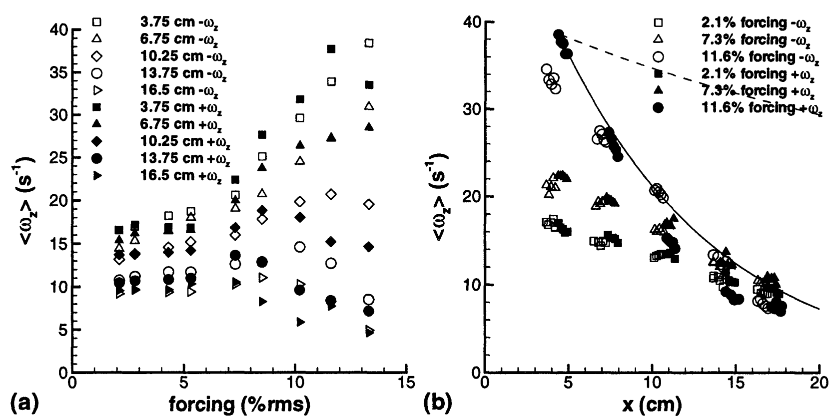


Figure 4: (a) Effect of forcing on peak vorticity magnitude at several downstream locations. (b) Variation of vorticity magnitude for 3 forcing amplitudes with respect to streamwise location. The solid line indicates the decay expected in the 11.6% forcing case solely due to negative stretching while the dashed line indicates the decay solely due to viscous diffusion.

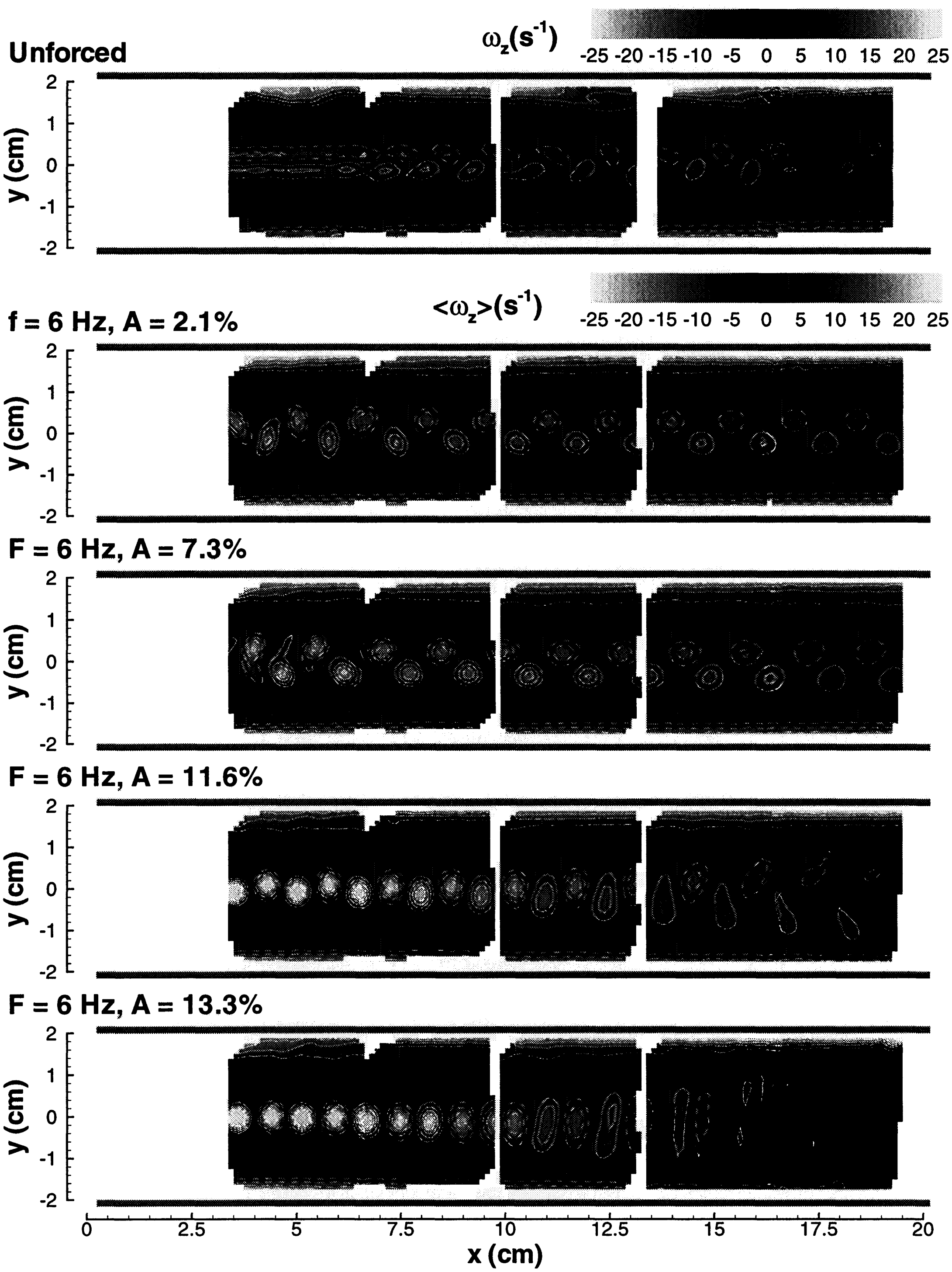


Figure 3: Spanwise vorticity field in forced and unforced wake at an arbitrary phase in the forcing cycle. The unforced wake data are instantaneous realizations and the forced wake data are phase-averaged results. Contour lines are spaced at $\pm 5\ s^{-1}$, $\pm 10\ s^{-1}$, ..., $\pm 25\ s^{-1}$. Dashed lines indicate negative vorticity.

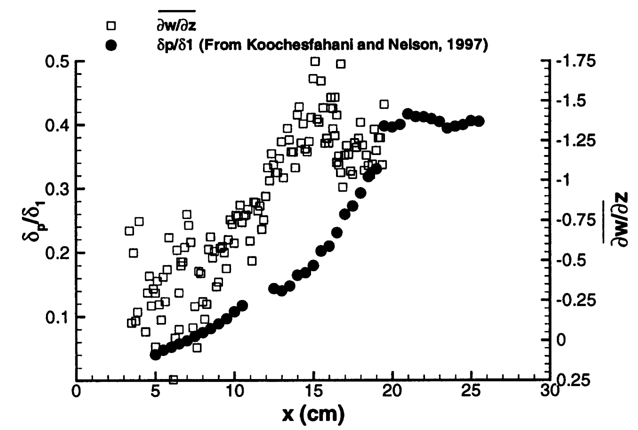


Figure 5: Comparison of mean ∂w/∂z in 11.6% forcing case and δ_p/δ_1 from Koochesfahani and Nelson (1997).

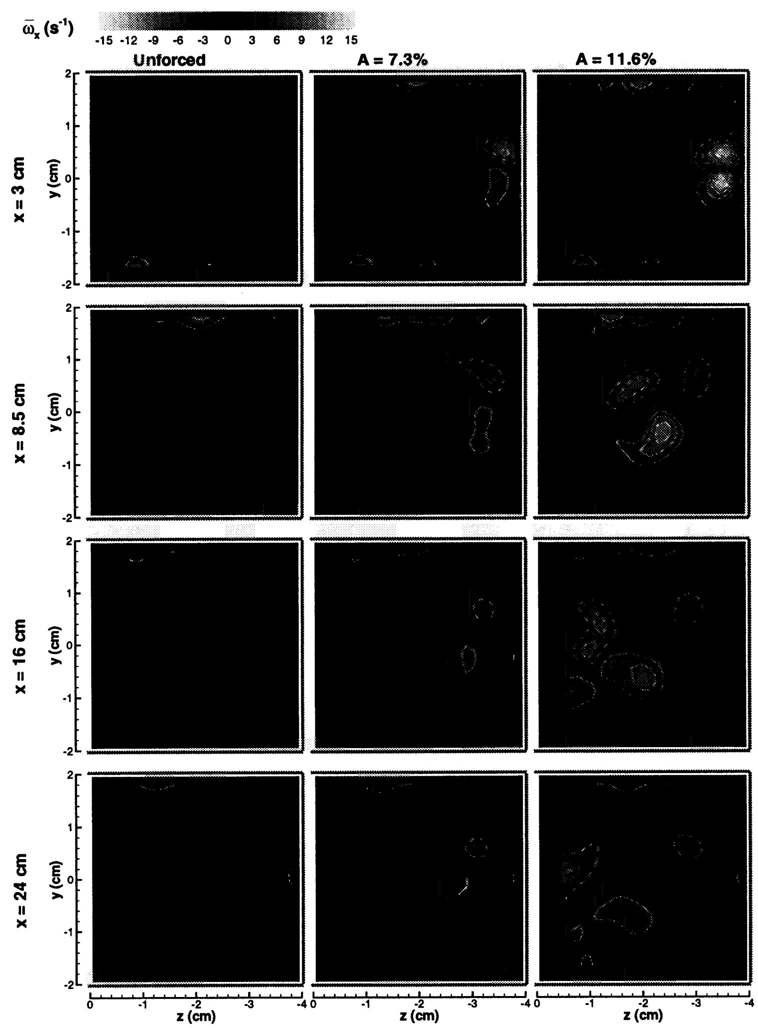


Figure 6: Mean streamwise vorticity field for the unforced, 7.3%, and 11.6% forcing cases at four downstream locations. Contour lines are spaced at ± 3 s^{-1}, ± 6 s^{-1}, ..., ± 15 s^{-1}. Dashed lines indicate negative vorticity.