SUBGRID SCALE MODELING OF TURBULENCE CONSIDERING THE EFFECT OF EXTERNAL FORCE

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ABSTRACT

The objective of our research is to develop the Subgrid-Scale (SGS) model in which external force is explicitly considered and to examine its validity in an actual numerical simulation. To begin with, we derived transport equations of SGS stress and scalar flux with external force following the generalized filtering operation proposed by Germano. Then on the assumption that the production term including the effect of external force was dominant among other terms, the proposed models were obtained as the product of the time scale to the production term. A model parameter included in the proposed models was determined by the dynamic procedure proposed by Germano et al. The proposed models are applied to the simulation of two types of flow with external force having the same geometry : one is the unsteady stratified and the other is the rotating turbulent channel flow. The predicted results of the proposed model shows better agreement with DNS data than the model without the effect of external force.

I. INTRODUCTION

In the field of mechanical, environmental or geophysical fluids dynamics, external force such as Coriolis force due to system rotation or buoyancy due to density variance often affects the turbulence intensively and consequently characteristic eddy structures appear in the flow field, which is different from the ones observed without external force. Compared with Reynolds averaged Navier-Stokes (RANS) method, Large Eddy Simulation (LES) seems to be a promising method to predict such flow field because basically larger (or Grid-Scale : GS) eddies, which have characteristic structures in each flow field and are affected more strongly by external force, are solved directly from the GS continuity and momentum equations while only smaller dissipative eddies are modeled. But considering the fact that mesh configurations of LES in practical problems are usually coarse, effect of external force on Subgrid-Scale (SGS) turbulence cannot be ignored. In fact, Shimomura (1997) reported the defect of Smagorinsky's model in rotating isotropic turbulence by coarse LES. In turbulence with buoyancy, external force works as the production of turbulence energy. Therefore by considering the energy balance of production and dissipation in SGS including the production due to buoyancy, SGS eddy viscosity coefficient of the ordinal Smagorinsky's model (Smagorinsky, 1963) can be modified (e.g., Eidson, 1985). But it seems that the isotropic eddy viscosity model essentially cannot predict the anisotropic structure of SGS turbulence produced by buoyancy . In turbulence with system rotation, effect of rotation is more difficult to be considered in SGS modeling than buoyancy case because Coriolis force does not produce SGS turbulence energy directly and works as re-distribution among each component of SGS normal stress. These facts drives us to the intensive requirement of some unified and preferably simple method to properly consider the effect of external force on the SGS modeling. Accordingly the purpose of our research here is to develop a SGS model considering the effect of external force and to investigate its validity in an actual numerical simulation.

The strategy of our modeling is the extension of the one proposed by Yoshizawa et al.(1996). We first derived transport equations of SGS stress and scalar flux with external force following the formulation of the generalized filtering operation proposed by Germano (1992). Then on the assumption that production terms are dominant among other terms, SGS model is obtained as the product of the production terms including external force and some time scale of SGS turbulence. The scale similarity concept proposed by Bardina (1983) was used for the modeling of SGS turbulence energy. The detail of the procedure as well as the governing equations are mentioned on Sec. II. Numerical method adopted in this study is briefly explained on Sec. III. We began with the simple channel flow with passive scalar without any external force to examine the validity of the SGS stress and flux model used. The results of the comparison are described in Sec. IV. Then, as the core of this study, the proposed models are tested in the unsteady stratified turbulent channel flow in case of buoyancy as external force and in the rotating channel flow in which Coriolis force is external force. The validity of our proposed model in these flow fields is also shown on Sec. IV. Finally some concluding remarks are addressed on Sec. V.

II. GOVERNING EQUATIONS

Transport Equations of the SGS Stress and SGS scalar flux

In LES, the concept of spacial filtering is essential. The spatially filtered velocity is defined by

$$\bar{u}_i(x,t) = \int_{-\infty}^{+\infty} G(x - x' ; \bar{\Delta})\, u_i(x,t)\, dx' \qquad (1)$$

where $\bar{\Delta}$ is a filter width and u_i is a velocity component for i direction. The momentum equation of incompressible flow for LES is easily obtained by conducting this filtering operation to the Navier-Stokes equation. Finally the filtering N-S eq. is given as

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial\, \bar{u}_i \bar{u}_k}{\partial x_k} = -\frac{\partial \bar{P}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_k \partial x_k} - \frac{\partial}{\partial x_k} \tau(u_i, u_k) + \bar{f}_i \qquad (2)$$

where $\bar{P}$ is the filtered pressure divided by constant density, ν is the kinematic viscosity and $\bar{f}_i$ is the filtered external force term that is important in this study. The transport equation of filtered scalar $\bar{\theta}$ is also easily obtained as follows,

$$\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial\, \bar{\theta}\, \bar{u}_k}{\partial x_k} = \alpha \frac{\partial^2 \bar{\theta}}{\partial x_k \partial x_k} - \frac{\partial}{\partial x_i} \tau(\theta, u_k) \qquad (3)$$

where α is the molecular diffusivity of scalar θ. Production of scalar is not considered here for convenience. In these two equations, the SGS stress tensor and SGS scalar flux are described following Germano's generalized filtering operation. The second and third moment described by the generalized filtering operation are given by

$$\tau(a,b) = \overline{a\,b} - \bar{a}\,\bar{b} \qquad (4)$$
$$\tau(a,b,c) = \overline{a\,b\,c} - \bar{a}\,\tau(b,c) - \bar{b}\,\tau(c,a) - \bar{c}\,\tau(a,b) - \bar{a}\,\bar{b}\,\bar{c}.$$

With a little tiresome transformation, the transport equations of the SGS stress tensor $\tau(u_i, u_j)$ and SGS scalar flux $\tau(u_i, \theta)$ are finally obtained from eqs. (2) and (3) as follows

$$\frac{\partial \tau(u_i,u_j)}{\partial t} + \frac{\partial}{\partial x_k}\{\bar{u}_k \tau(u_i,u_j)\}$$
$$= -T_{ij} + \Pi_{ij} - \varepsilon_{ij} \qquad (5)$$
$$- \tau(u_i,u_k)\frac{\partial \bar{u}_j}{\partial x_k} - \tau(u_j,u_k)\frac{\partial \bar{u}_i}{\partial x_k} + \tau(f_j, u_i) + \tau(f_i, u_j)$$

$$\frac{\partial}{\partial t}\tau(u_i,\theta) + \frac{\partial}{\partial x_k}\bar{u}_k \tau(u_i,\theta)$$
$$= -d_{i\theta} + \phi_{i\theta} - \varepsilon_{i\theta} \qquad (6)$$
$$- \tau(u_k,\theta)\frac{\partial \bar{u}_i}{\partial x_k} - \tau(u_i,u_k)\frac{\partial \bar{\theta}}{\partial x_k} + \tau(f_i,\theta)$$

where only production terms of each equations are shown in detail and other terms are denoted by symbol such as T_{ij} or $d_{i\theta}$ because in this study, only production term is important. The third moment of velocity or scalar is included in T_{ij} and $d_{i\theta}$. The important thing to be noted here is that we consider the modeling of only SGS Reynolds stress and flux term here and in this case, SGS stress and flux are given as,

$$\tau(u_i,u_j) \approx -\overline{u_i' u_j'}$$
$$\tau(u_i,\theta) \approx -\overline{u_i' \theta'} \qquad . (7)$$

The reason why we consider only SGS eddy interaction is that we focus on only the modeling of energy dissipation including external force here. It is known that as the model of energy dissipation born by SGS Reynolds stress, eddy viscosity type model adopted here seems to be a good candidate while the interaction of GS and SGS eddies, scale similarity model and its extension have been proposed by some researchers. The proposed models are easily extended by coupling the scale similarity model (e.g., Horiuti, 1997) as a result.

SGS stress modeling

Yoshizawa et al. (1996) derived the SGS stress model as the product of the production term of its transport equation and some turbulent time scale under the assumption that in SGS field, production of the stress is dominant among the others. We follow that strategy to model SGS stress including external force. With the assumption of the weak anisotropy of the SGS velocity field, SGS stress is expressed as the product of the production term of eq. (5) and time scale T, which is given as

$$\tau(u_i,u_j) - \frac{1}{3}\delta_{ij}\tau(u_k,u_k) \qquad (8)$$
$$\approx -\frac{2}{3}T_1\, \tau(u_k,u_k)\,\bar{S}_{ij} + T_2\{\tau(f_j,u_i) + \tau(f_i,u_j)\}$$

with

$$\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad . \quad (9)$$

In this equation, the second term on the right represents the effect of external force in the modeled SGS stress. It should be noted that further modeling is required for $\tau(u_k, u_k)$, $\tau(f_i, u_j)$, T_1 and T_2 in the equation on the right. For the time scales, followings would be two of the candidates,

$$T = C\,\bar{\Delta} / \sqrt{\tau(u_k,u_k)} \quad , \qquad (10)$$

$$T = C / \sqrt{2\,\bar{S}_{ij}\bar{S}_{ij}} \qquad . \qquad (11)$$

The first one was adopted by Yoshizawa et al. (1996). The second is used in this study in the sense of a numerical simulation because it does not include the length scale explicitly and consequently if dynamic procedure is adopted to determine model parameter, the ratio of *Test* and *Grid* filter as a parameter of numerical simulation does not explicitly appear in the procedure. We also confirmed the validity of eq. (11) to eq. (10) in the actual simulation of turbulent channel flow at low Reynolds number. The detail of these results are mentioned in Tsubokura et al. (1997). In this study, we suppose that T_1 and T_2 are the same to restrict the number of the model parameter included in the model to one for numerical convenience. We agree that there is no theoretical reason of considering these two time scale is the same. The extension of two parameter dynamic procedure has been developed by a few researchers (e.g., Horiuti, 1997). It is not difficult to extend our proposed model here to two parameter procedure but which remain in future work. For the modeling of $\tau(u_k, u_k)$ and $\tau(f_i, u_j)$ included in eq.(8) on the right, the scale similarity model of Bardina (1983) is adopted which is given as

$$\tau(u_k, u_k) \approx (\bar{u}_k - \bar{\bar{u}}_k)(\bar{u}_k - \bar{\bar{u}}_k) \qquad (12)$$

$$\tau(f_i, u_j) \approx (\bar{f}_i - \bar{\bar{f}}_i)(\bar{u}_j - \bar{\bar{u}}_j) \qquad . \qquad (13)$$

What is to be noticed here is that we consider only the SGS Reynolds stress modeling mentioned above, therefore, we also model only the SGS interaction part for $\tau(u_k, u_k)$ and $\tau(f_i, u_j)$ appearing on the right of eq. (8). The final form of SGS stress including the effect of external force is described as follows,

$$\begin{aligned} &\tau(u_i, u_j) - \frac{1}{3}\delta_{ij}\tau(u_k, u_k) \\ &= C / \bar{S} \lbrack -\frac{2}{3}(\bar{u}_k - \bar{\bar{u}}_k)(\bar{u}_k - \bar{\bar{u}}_k)\bar{S}_{ij} \\ &+ \{(\bar{f}_j - \bar{\bar{f}}_j)(\bar{u}_i - \bar{\bar{u}}_i) + (\bar{f}_i - \bar{\bar{f}}_i)(\bar{u}_j - \bar{\bar{u}}_j)\} / \bar{S} \end{aligned} \qquad (14)$$

where

$$\bar{S} = \sqrt{2\bar{S}_{ij}\bar{S}_{ij}} \qquad . \qquad (15)$$

This model can be used to the flow with any types of external force only if the external force f_i is expressed as the linear function of the quantity of flow field.

SGS Flux Modeling

Following the procedure used for SGS stress modeling, SGS flux is modeled as follows,

$$\begin{aligned} &\tau(u_i, \theta) \qquad (16) \\ &\approx C_\theta / \bar{S} \{ -\frac{1}{3}(\bar{u}_k - \bar{\bar{u}}_k)(\bar{u}_k - \bar{\bar{u}}_k)\frac{\partial \bar{\theta}}{\partial \bar{x}_i} + (\bar{f}_i - \bar{\bar{f}}_i)(\bar{\theta} - \bar{\bar{\theta}}) \} \end{aligned}$$

It should be noted that eq. (16) was obtained under the assumption of weak anisotropy of the SGS velocity field, that is, $\tau(u_i, u_j) \sim 1/3\delta_{ij}\tau(u_k, u_k)$ and $\tau(u_i, \theta) \sim 0$ is used on the right of eq. (6).

Dynamic SGS model

In this study, the dynamic procedure developed by Germano et al. (1991) with the modification of Lilly (1992) is adopted to determine the model parameter C included in eq. (14). The parameter for SGS scalar flux C_θ in eq. (16) is also determined by the same method (e.g., Moin et al., 1991). Hereafter, the *Test* filtering operation required in the method is expressed as *over-tilde* (~)

III. NUMERICAL METHOD

Discretization of Governing Equations.

Governing equations including a SGS model were discretized based on the finite difference method considering future application to engineering problems. The newly developed 4th order accurate finite difference schemes proposed by Morinishi (1995) was adopted in which both momentum and kinetic energy are conserved at the 4th order accuracy in the discretized sense in a staggered grid system.

A semi-implicit time marching algorithm was adopted in which normal wall direction of the diffusion terms were only implicitly solved with the Crank-Nicolson scheme while the third order Runge-Kutta scheme was used for the other terms.

Discretization of Filtering Operation.

The filtering operation given by eq. (1) must be discretized in this study for the double filtering of GS included in eqs. (14) and (15) as well as the dynamic procedure. The discrete filtering operation for finite difference method in 4th order accuracy is given by

$$\bar{u}_i = u_i + \frac{\bar{\Delta}^2}{24}\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + O(\Delta^4) \qquad (17)$$

where $\bar{\Delta}$ is the filter width and h is the grid width in FDM. The important point to be noted in this equation is that the ratio of $\bar{\Delta}$ and h is the parameter to be determined *a priori*. We have adopted $\bar{\Delta}/h = \sqrt{2}$ and $\tilde{\Delta}/h = \sqrt{6}$ for *Grid* and *Test* filtering respectively considering the fact that $\tilde{\bar{\Delta}}/\bar{\Delta} = 2$ is proposed by Germano et al. (1991) and that $\bar{\Delta}^2 + \tilde{\Delta}^2 = \tilde{\bar{\Delta}}^2$ is true in Gaussian filter. In this case $\tilde{\bar{\Delta}}/h = 2\sqrt{2}$. These optimized values have been obtained from the numerical simulation of the turbulent channel flow at low Reynolds number. The detail is referred in Tsubokura et al. (1997)

IV. RESULTS OF NUMERICAL SIMULATION

All flow geometry of the simulation conducted in this study is the simple turbulent channel flow. Therefore we adopted following notation for the coordinate as x or x_1, y or x_2, z or x_3 for streamwise, normal-wall, spanwise direction respectively. In all simulation, the total mesh number of only 32x65x32 is used to consider the practical or engineering application. Periodic boundary condition is adopted for streamwise and spanwise direction while no-slip condition is used on the wall.

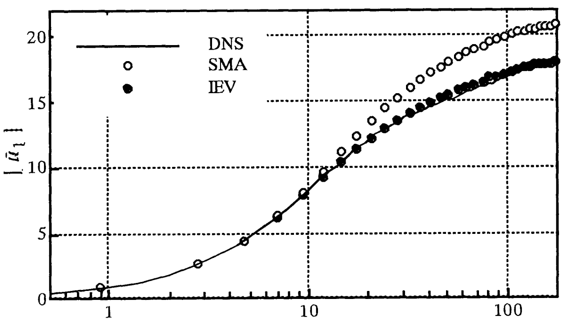

Fig. 1 Mean velocity profiles of the channel flow at $Re\tau$=180

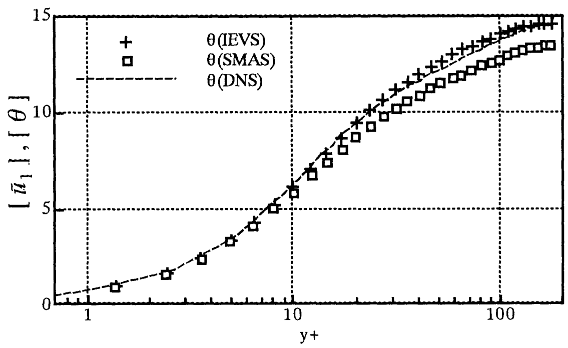

Fig. 3 Mean passive scalar profiles of channel flow, $Re\tau$=180, Pr=0.7

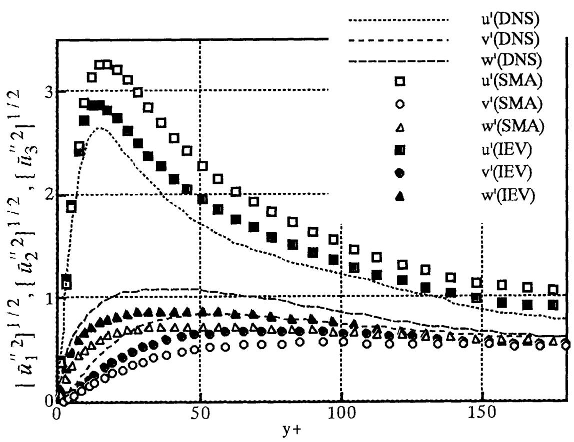

Fig. 2 GS turbulent intensity of a channel flow at $Re\tau$=180

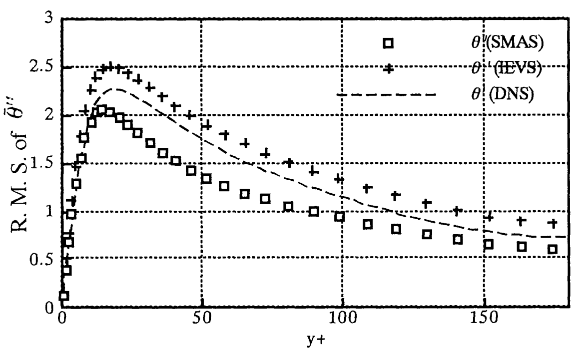

Fig. 4 R.M.S. of GS passive scalar fluctuation at $Re\tau$=180, Pr=0.7

Turbulent channel flow with passive scalar

To investigate the fundamental validity of the eddy viscosity model in eq. (14) to the Smagorinsky's model, we first mention the results of the turbulent channel flow without external force at $Re\tau$=180 normalized by friction velocity and channel half-wodth (δ) with passive scalar at Pr=0.7. In this case, f in eqs. (14) and (16) are given as zero. The length of the computational region for x, y and z direction is set to $4\pi\delta$, 2δ and $4\pi\delta/3$ respectively. Consequently, the grid width for streamwise and spanwise direction normalized by wall-unit is given as Δx^+=70.7 and Δz^+=23.5.

Fig. 1 and 2 indicate the mean velocity profiles and GS turbulent intensity profiles. In these figures, "IEV" and "SMA" means the results of the proposed Isotropic Eddy Viscosity model given as eq. (14) and simple SMAgorinsky model (Smagorinsky, 1963) given as

$$\tau(u_i, \bar{u}_i) = -2C\bar{\Delta}^2\bar{S}\,\bar{S}_{ij} \quad . \tag{18}$$

"DNS" is the result of Direct Numerical Simulation obtained by Kim et al. (1987). The typical drawbacks of Smagorinsky's model with dynamic procedure at this low Re case are the over-prediction of log-law region and the streamwise intensities. It is clearly observed that such problems are mitigated by "IEV" and all profiles of GS turbulent intensity are improved by "IEV". It is somehow disappointing to say that even "IEV" still have some problem on the prediction of turbulent intensity compared to the DNS results. We can say that such defective prediction seems to be the fundamental problem of the isotropic eddy viscosity model and some improvement is reported by coupling the scale similarity model with the eddy viscosity model (e.g., Horiuti, 1997).

Fig. 3 and 4 indicate the mean and RMS profiles of a scalar in two types of model. The condition of scalar is that it uniformly input within the fluid and then removed at both walls. DNS data of scalar statistics were obtained by Horiuti (1991). It should be noted here that in both cases, "IEV" model is adopted for SGS stress model. The tested SGS scalar flux model are the one given in eq. (16) expressed as "IEVS" and the one obtained as the extension of Smagorinsky's model ("SMAS" in the figures) described as,

$$\tau(u_i, \theta) = -C\bar{\Delta}^2\bar{S}\frac{\partial\,\theta}{\partial\bar{x}_i} \tag{19}$$

These two figures shows that "IEVS" predicts better results than "SMAS". From these four figures, we can say that "IEV" and "IEVS" have advantage over Smagorinsky's in this low Reynolds number flow.

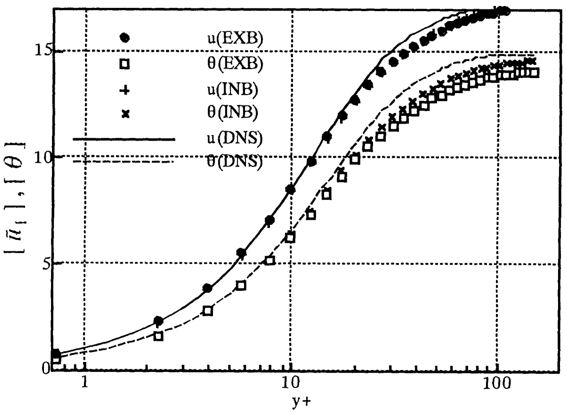

Fig. 5 Mean velocity and temperature profiles of unstratified channel flow, *Reτ*=150, *Pr*=0.7, *Gr*=1.3e⁻⁶

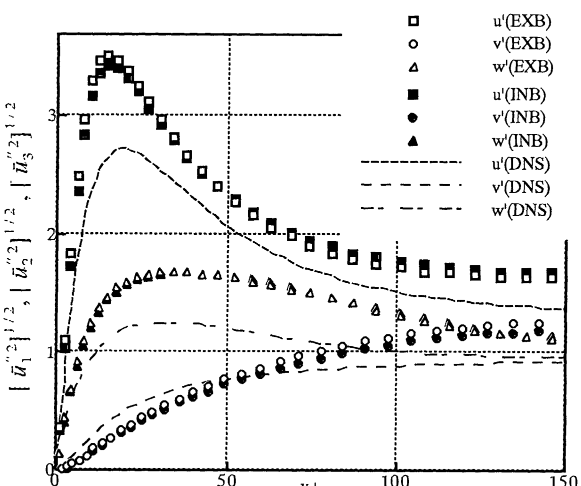

Fig. 6 GS turbulent intensity of unstratified channel flow, *Reτ*=150, *Pr*=0.7, *Gr*=1.3e⁻⁶

Unstably stratified turbulent channel flow

Now we attempt to extend the test of our proposed model to the flow with buoyancy as external force. The flow we simulated here is the unstably stratified turbulent channel flow at Re_τ=150, Pr=0.7 and $Gr=g\beta\Delta T(2\delta)^3/\nu^2$=1.3x10^6. Here we adopted the Boussinesq approximation for buoyancy and accordingly incompressible Navier-Stokes equations are used. The length of the computational region for x, y and z direction is given as $6\pi\delta$, 2δ and $2.4\pi\delta$ respectively and in this case, the grid width for streamwise and spanwise direction normalized by wall-unit is given as Δx^+=88.3 and Δz^+=35.3. The DNS results were obtained by Iida and Kasagi (1997). The two walls of the channel are assumed to have different but constant temperatures to make unstable density stratification. In this case, the vertical fluid motion of thermal plumes will appear on the flow field, which change the transport mechanism of turbulence drastically.

The tested models for SGS stress and scalar flux are the model with buoyancy effect (INcluding Buoyancy : INB) given as

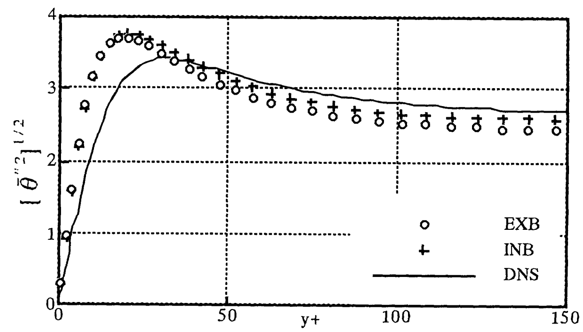

Fig. 7 Root mean square of temperature fluctuation of unstratified channel flow, *Reτ*=150, *Pr*=0.7, *Gr*=1.3e⁻⁶

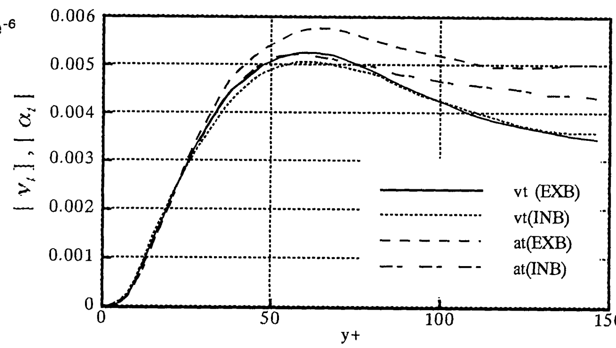

Fig. 8 SGS eddy viscosity coef. and diffusivity coef., *Reτ*=150, *Pr*=0.7, *Gr*=1.3e⁻⁶

$$\begin{aligned}
&\tau(u_i,u_j)-\frac{1}{3}\delta_{ij}\tau(u_k,u_k)\\
&=C l\,\tilde{S}\{-\frac{2}{3}(\bar{u}_k-\bar{\bar{u}}_k)^2\overline{S}_{ij}+\beta g_i(\bar{\theta}-\bar{\bar{\theta}})(\bar{u}_j-\bar{\bar{u}}_j)\}\\
&\tau(u_i,\theta)\\
&=-\frac{1}{3}C_s l\,|\tilde{S}|\{(\bar{u}_k-\bar{\bar{u}}_k)^2\frac{\partial\bar{\theta}}{\partial x_i}+\beta g_i/\tilde{S}\}(\bar{\theta}-\bar{\bar{\theta}})(\bar{\theta}-\bar{\bar{\theta}})\}
\end{aligned} \quad (20)$$

and ones without buoyancy term in eq. (20) (EXcluding Buoyancy : EXB).

We can observe the improved results of INB compared to the EXB in mean temperature and GS r.m.s. of temperature fluctuation especially at the region away from the wall, which is shown in Fig. 5 and Fig. 7. It is regrettable to say that only slight improvement of GS streamwise turbulent intensity is observed and the results of INB and EXB in mean velocity is almost the same. Such tendency is also identified in the profile of SGS eddy viscosity and diffusivity coefficient, which is given in Fig. 8 where only slight difference is observed in SGS eddy viscosity while SGS diffusivity of scalar has moderate difference especially away from the wall.

Rotating turbulent channel flow

Secondly we consider the Coriolis force as external force.

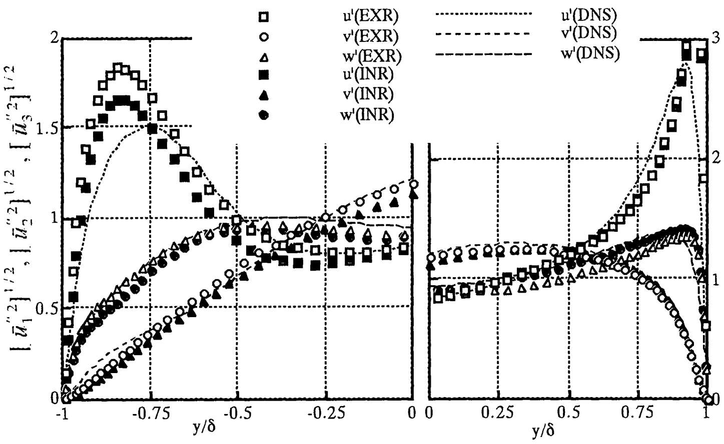

Fig. 9 GS turbulent intensity of rotating channel flow, *Reτ*=150, *Roτ*=2.5 (left: suction side, right: pressure side)

The flow simulated here is the turbulent channel flow at *Reτ*=150 in rotating reference frames on the axis for spanwise direction. The rotation number normalized by friction velocity, $Ro_\tau = 2\,\Omega\,\delta\,/\,u_\tau$, is set to 2.5, which is the same as the DNS conducted by Nishimura and Kasagi (1996). The length of the computational region for x, y and z direction is $5\pi\delta$, 2δ and $4\pi\delta/3$ respectively. Accordingly the grid width for streamwise and spanwise direction normalized by wall-unit is given as Δx^+=73.6 and Δz^+=29.4. It is known that in this flow configuration, stabilization on one side of the wall (suction side) and the destabilization on the other side (pressure side) emerge. The Coriolis force in rotating reference frames on the axis for x_3 direction is described as

$$\bar{f}_i = -2\,\varepsilon_{3ki}\,\Omega_3\,\bar{u}_k \quad (21)$$

where Ω_3 is the angular velocity of the system. The proposed SGS stress model including the effect of Coriolis force is obtained by just substitute eq. (21) into eq. (14), which is given as INR (INcluding Rotating effect). We compare INR model to the model without the effect of Coriolis force, which is expressed as EXR (EXcluding Rotating effect) in Fig. 9.

We can observe the improvement of INR on EXR model for streamwise GS intensity near the suction side.

V. CONCLUDING REMARKS

SGS stress and scalar flux models considering the effect of external force were proposed and tested in two types of the flow affected by external force: the unsteady stratified turbulent channel flow and the rotating turbulent channel flow. In both case, we can observe the improved results compared to the model without external effect.

The slight improvement of the proposed model on some statistics may be due to the low Reynolds number of the flows tested here, that is, the weakness of SGS effect on GS field is not strong at this low *Re*. Further investigation such as in high *Re* case is remaining in future work.

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