WALL EFFECTS ON THE TURBULENCE STRUCTURE AND ON THE VOID FRACTION DISTRIBUTION IN BUBBLY FLOWS

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ABSTRACT

We apply a two-fluid model with a second order closure of the turbulence in the simulation of wall bounded bubbly flows. We analyse the interfacial effects on the turbulence structure and we evaluate the role of the turbulence and of the interfacial forces on the void fraction distribution near the wall. The simulation of the pipe bubbly flow in micro-gravity condition shows that the turbulent contributions issued from the added mass force play an important role in the phase distribution phenomenon. In normal gravity, the simulations of boundary layer and pipe bubbly flows allow to adjust the lift force model in which we take into account the wall interaction effect.

INTRODUCTION

Many bubbly flows experiences show that the presence of the dispersed phase alters considerably the liquid turbulence structure (Lance et al, 1991, Moursali et al, 1995, Serizawa et al, 1986). In the other hand, the turbulence of the liquid phase was pointed to have an important role in the phase distribution (Drew and Lahey, 1982).

The elaboration of general models allowing to predict the two-phase turbulent flows (average and fluctuating velocities of both phases and their phase fraction) represents an actual challenge. However, in spite of the progress achieved in bubbly flow modelling (Lee et al, 1990, Lance et Lopez de Bertonado, 1992) some important difficulties subsist; in particular the ability to predict the phase distribution remains limited by the inadequate modelling of the turbulence and of the interfacial forces. The model presented here attempts to improve the ability of two-fluid flow models to predetermine bubbly flows with moderate void fraction by laying emphasis on two aspects of the interaction between phases. First we take into account the turbulent correlations issued from the added mass in the expression of the force exerted by the liquid on the bubbles: this turbulent contribution, which is customary ignored, is proving to be important in the phase distribution phenomena (Chahed et Masbernat, 1998b); and second we develop a turbulence model adapted to bubbly flows in order to describe accurately the Reynolds stress tensor in the liquid. The model is applied to the simulations of wall bounded bubbly flows and the numerical results are confronted to the experimental data of the boundary layer (Moursali et al, 1995) and of pipe bubbly flows (Kamp et al, 1994, Serizawa et al, 1986).

I. AVERAGE MASS AND MOMENTUM BALANCE EQUATIONS IN BUBBLY FLOW

We note χ_G the characteristic function of the gas phase ($\chi_G(M,t)=1$ if the point M is in the gas and 0 if not) and $<\Phi>$ the average field of Φ, $<>$ is an averaging operator that verifies the Reynolds conditions. The void fraction α, the mean field $\overline{\overline{\Phi_G}}$ and the fluctuating field Φ_G' in the gas phase are given by:

$$\alpha = <\chi_G> \tag{1}$$

$$\alpha\overline{\overline{\Phi_G}} = \chi_G\Phi \tag{2}$$

$$\chi_G \Phi = \alpha \overline{\overline{\Phi_G}} + \chi_G \Phi'_G \quad (3)$$

In the same manner, the mean and the fluctuating fields $\overline{\overline{\Phi_L}}$ and Φ'_L in the liquid phase, are given by:

$$(1-\alpha)\overline{\overline{\Phi_L}} = < (1-\chi_G)\Phi > \quad (4)$$

$$(1-\chi_G)\Phi = (1-\alpha)\overline{\overline{\Phi_L}} + (1-\chi_G)\Phi'_L \quad (5)$$

We note $\overline{\overline{\mathbf{u}}}$ and $\overline{\overline{\mathrm{p}}}$ the average velocity and pressure fields and $\mathbf{g}$ the gravity acceleration. For stationary incompressible bubbly flows without mass transfer ($\rho = \text{const}$), the averaged balance equations of mass and momentum in the liquid and in the gas are:

$$\nabla \bullet (1-\alpha)\overline{\overline{\mathbf{u}_L}} = 0 \quad (6)$$

$$\nabla \bullet \alpha \overline{\overline{\mathbf{u}_G}} = 0 \quad (7)$$

$$(1-\alpha)\rho_L \frac{D}{Dt}\overline{\overline{\mathbf{u}_L}} = -\nabla \overline{\overline{p_L}} - \nabla \bullet ((1-\alpha)\rho_L \overline{\overline{\mathbf{u}'_L \mathbf{u}'_L}}) + (1-\alpha)\rho_L \mathbf{g} \quad (8)$$

$$0 = -\alpha_G \nabla \overline{\overline{p_L^{(0)}}} + \mathbf{M}_G \quad (9)$$

with $\frac{D}{Dt} = \frac{\partial}{\partial t} + (\overline{\overline{\mathbf{u}_L}}.\nabla)$

In the equation of the momentum balance, we neglect the acceleration and the weight of the gas as compared to the force exerted by the liquid on the bubbles $\rho_G << \rho_L$; so the total force exerted on the bubbles is zero as indicated in equation (9). This force contains the non disturbed flow action (pressure term or Tchen force) and the interfacial term $\mathbf{M}_G = < \chi_G \mathbf{f}_p >$ where $\mathbf{f}_p$ is the density of the force due to the perturbed flow action.

Assuming the spatial homogeneity of the first and the second order moments over the scale of the bubble, the force density is expressed by:

$$\mathbf{f}_P = \frac{1}{\vartheta_B} \oiint_{\partial\vartheta_B} \sigma_L .\mathbf{n} dS = \overline{\nabla.\sigma_L^{(1)}}^{(\vartheta_B)} \quad (10)$$

where $\sigma_L^{(1)}$ designates the stress field due to the perturbed flow, ϑ_B and $\partial\vartheta_B$ are respectively the volume and the boundary of the bubble and $\overline{}^{(\vartheta_B)}$ is an average operator over the volume of the bubble.

The formulation of the momentum interfacial exchange set many questions especially concerning the averaging of the fluctuating terms issued from of the drag, added mass and lift forces. The common method consists to keep the contributions due the average velocities fields of the liquid and the gas phases while the turbulent contributions of the interfacial force are ignored or eventually expressed via a supplementary dispersion term proportional to the void fraction gradient (Lance and Lopez de Bertonado, 1992). We will show the insufficiency of these formulations and we propose to take into account the turbulent correlations issued from the added mass force; we have proved that t these turbulent correlations, which contains turbulent correlations in the liquid and in the gas, are important in the phase distribution phenomenon (Chahed and Masbernat, 1998b). The interfacial momentum transfer term employed here is:

$$\mathbf{M}_G = -\frac{3}{4}\rho_L \frac{C_D}{d}\left\|\overline{\overline{\mathbf{u}_R}}\right\|\overline{\overline{\mathbf{u}_R}} - 2\rho_L C_L (1 - f_{LP}(y^{**}))\overline{\overline{\omega_L}} \times \overline{\overline{\mathbf{u}_R}} - \rho_L C_A (\frac{d}{d}\overline{\overline{\mathbf{u}_G}} - \frac{D}{D}\overline{\overline{\mathbf{u}_L}}) - \rho_L C_A \frac{1}{\alpha}\nabla.\alpha(\overline{\overline{\mathbf{u}'_G \mathbf{u}'_G}} - \overline{\overline{\mathbf{u}'_L \mathbf{u}'_L}}) \quad (11)$$

Where $\frac{d}{dt} = \frac{\partial}{\partial t} + (\overline{\overline{\mathbf{u}_G}}.\nabla)$) and $\overline{\overline{\mathbf{u}_R}}$ is the relative velocity of the bubbles defined by:

$$\alpha \overline{\overline{\mathbf{u}_R}} = < \chi_G (\mathbf{u}_G - \mathbf{u}) > = \alpha(\overline{\overline{\mathbf{u}_G}} - \overline{\overline{\mathbf{u}}}) - < \chi_G \mathbf{u}' > \quad (12)$$

The interfacial force (11) includes respectively the drag force (drag coefficient C_D), the lift force (coefficient C_L) that is expressed with a modified lift coefficient taking into account an eventual wall interaction effect and the added mass force (coefficient C_A) that includes the average and turbulent contributions. The other turbulent contributions issued from the drag and lift forces are omitted assuming first that the fluctuation of the slip velocity can be taken into account through an eventual modification of the average drag coefficient and second that the liquid velocity and vorticity fluctuations are weakly correlated.

The last term in the relative velocity expression, equation (12), represents a correlation between the continuous phase velocity fluctuation and the instantaneous phase distribution and is modelled following Simonin et al (1989) as a drifting velocity which takes into account the dispersion effect due to bubbles transport by the turbulent fluid motion.

II. TRANSPORT MODEL OF THE REYNOLDS STRESS TENSOR IN BUBBLY FLOWS

The turbulent correlations of the dispersed phase $\overline{\overline{\mathbf{u}_G^{'}\mathbf{u}_G^{'}}}$ are related to liquid turbulent stress tensor through a turbulent dispersion model based on the Tchen theory in homogenous flow. The turbulent stress tensor of the continuous phase $\overline{\overline{\mathbf{u}_L^{'}\mathbf{u}_L^{'}}}$ is computed using a second order closure of the Reynolds stress tensor in bubbly flows, (Chahed and Masbernat, 1998a). In this model, the Reynolds stress tensor in the liquid is separated into two parts: an irrotationnal part $\overline{\overline{\mathbf{u}_L^{'}\mathbf{u}_L^{'}}}^{(S)}$ induced by the bubbles displacements and controlled by the added mass force and a turbulent part $\overline{\overline{\mathbf{u}_L^{'}\mathbf{u}_L^{'}}}^{(T)}$ produced by the gradient of the mean velocity which also contains the turbulence generated in the bubbles wakes; we propose a modelled transport equation for each part as follows:

$$\frac{D}{Dt}\overline{\overline{\mathbf{u}_L^{'}\mathbf{u}_L^{'}}}^{(S)} = \mathrm{Diff}(\overline{\overline{\mathbf{u}_L^{'}\mathbf{u}_L^{'}}}^{(S)}) + \frac{3}{20}\frac{D}{Dt}\alpha\left\|\overline{\overline{\mathbf{u}_R}}\right\|^2\delta + \frac{1}{20}\frac{D}{Dt}\alpha\overline{\overline{\mathbf{u}_R\mathbf{u}_R}} \quad (13)$$

$$\frac{D}{Dt}\overline{\overline{\mathbf{u}_L^{'}\mathbf{u}_L^{'}}}^{(T)} = \mathrm{Diff}(\overline{\overline{\mathbf{u}_L^{'}\mathbf{u}_L^{'}}}^{(T)}) - 2\mathrm{sym}\left[\overline{\overline{\mathbf{u}_L^{'}\mathbf{u}_L^{'}}}^{(T)} \bullet \nabla\overline{\overline{\mathbf{u}_L}}\right] + \Phi - \varepsilon\delta \quad (14)$$

The transport equation of the pseudo-turbulent part, equation (13), expresses a diffusif transport and a production-redistribution mechanism related to the added mass force. In the transport equations of the turbulent part, equation (14), it is assumed that the interfacial production of the turbulent energy and its dissipation rate are balanced in the bubbles wakes. The dissipation is thus identified to the isotropic dissipation in the small scales resulting from the energy cascade and the transport equation has a same form as in single-phase flow.

The diffusion and redistribution terms in equation (14) are modified in order to take into account the interfacial effects. In the model of the diffusion term we introduce a supplementary turbulent transport par the bubbles with the characteristic time scale τ_b; so the single-phase model of Launder et al (1975) is "generalised" in two-phase bubbly flows in the form:

$$\mathrm{Diff}(\Psi) = \frac{C_{S\Psi}}{1-\alpha}\nabla\left[(1-\alpha)(\tau_t\overline{\overline{\mathbf{u}^{'}\mathbf{u}^{'}}}^{(T)} + \tau_b\overline{\overline{\mathbf{u}^{'}\mathbf{u}^{'}}}^{(S)})\nabla\Psi\right] \quad (15)$$

$$\text{with } \tau_t = \frac{\mathrm{trace}(\overline{\overline{\mathbf{u}^{'}\mathbf{u}^{'}}}^{(T)})}{2\varepsilon}, \qquad \tau_b = C_R\frac{d}{\left\|\overline{\overline{\mathbf{u}_R}}\right\|}$$

It should be noted that the diffusion effect associated to the bubbles motions $\tau_b\overline{\overline{\mathbf{u}^{'}\mathbf{u}^{'}}}^{(S)}$ generalises Sato et al (1981) formulation of the turbulent viscosity induced by the bubbles.

In a same way as in single-phase flow, the redistribution term $\Phi = \Phi^{(L)} + \Phi^{(NL)}$ is spilt into two-part, a linear part $\Phi^{(L)}$ and a non-linear part $\Phi^{(NL)}$. In the non-linear part $\Phi^{(NL)}$, we introduce a supplementary stretching effect produced by the bubble displacement with the time scale τ_b in order to represent the tendency to the more isotropy observed in bubbly flows (Lance et al 1991).

$$\Phi^{(NL)} = -C_1(\tau_t^{-1} + \alpha\tau_b^{-1})\left[\overline{\overline{\mathbf{u}^{'}\mathbf{u}^{'}}}^{(T)} - \frac{1}{3}\mathrm{trace}(\overline{\overline{\mathbf{u}^{'}\mathbf{u}^{'}}}^{(T)})\right] \quad (16)$$

The linear part of the redistribution term $\Phi^{(L)}$ is modelled as in single-phase and the Launder et al (1975) model is adopted. We also take into account the wall effect on the redistribution mechanism using a similar single-phase phase model. The transport equation of the dissipation rate ε is also the same as in single-phase flow where the diffusion term is modelled according to (15).

III NEAR-WALL ANALYSIS IN BUBBLY FLOW

The wall effect in bubbly flow is double: in one hand, in a same way as in single phase flow, we have to interpret the wall effect on the average and turbulent kinematic fields according to wall laws in order to define the boundary conditions of the model; in the other hand we have to express the wall effect on the force exerted by the continuous phase on the bubbles.

III.1 Wall effects on the average and turbulent kinematics fields

In their experience of bubbly boundary layer, Moursali et al (1995) have proved that the liquid average velocity have logarithmic profiles near the wall with the constants χ and

b that are different from the single-phase constants χ_0 and b_0. We note $\overline{\overline{u}}$ the average longitudinal velocity, u_* the friction velocity and y the distance from the wall. The logarithmic wall law is:

$$\frac{\overline{\overline{u}}}{u_*} = \frac{1}{\chi} Ln \frac{u_* y}{\nu} + b \quad (17)$$

The liquid momentum equation (4) has the same form as in single phase flow and the difference between the constants of the single phase and the two-phase logarithmic profiles is certainly due to the bubbles effect on the turbulent viscosity in the near wall flow region.

In the logarithmic near wall zone, the turbulent viscosity in bubbly flow is $\nu_t = \chi u_* y$ while the model of Sato et al (1981) gives, in a same manner as our model, an expression of the turbulent viscosity in the form:

$$\nu_t = \chi_0 u_{*0} y + C_b \alpha d \left| \overline{\overline{u_G}} - \overline{\overline{u}} \right| \quad (18)$$

where u_{*0} is the friction velocity of the single-phase flow with the same average velocity gradient. We thus obtain the following relation between the single-phase and two-phase constants χ and χ_0:

$$\frac{\chi}{\chi_0} = 1 + a(1 + 0.5a) + O(a^4) \quad (19)$$

where $a = \frac{C_b \alpha d}{2 y_p} \frac{\overline{\overline{u_R}}}{\chi_0 u_*}$

If we assume a linear distribution of the void fraction between the wall (where the void fraction is zero) and the void fraction peaking, the parameter "a" is thus:

$$a \approx C_b \alpha_p \frac{\overline{\overline{u_R}}}{\chi_0 u_*} \quad (20)$$

Where α_p is the value of the void fraction peaking located at the distance $y_p^{**} = \frac{2 y_p}{d} \approx 1$, (Moursali et al, 1995)

The determination of the second constant b of the logarithmic profile is founded upon the assumption that the thickness S_0 of the viscous sub-layer is not modified in two-phase bubbly flows, (Moursali, 1995). Thus we deduce the following relation:

$$b = b_0 + (1 - a(1 + 0.5a))(S_0 - b_0) \quad (21)$$

For a given boundary layer bubbly flow, the equations (19), (20) and (21) indicate that the constants of the logarithmic law in bubbly flow depends on the amplitude of the void fraction peaking near the wall. This outcome is in perfect concordance with the experimental results of Moursali et al (1995).

III.2 Wall effect on the interfacial force

The interaction between the bubbles and the wall introduces a modification of the force exerted on the bubbles particularly on the lift force: the drainage of the liquid film between the bubble and the wall brings about a repulsive force that reduces the averaged lift force; We note $F_L = F_{L0} + F_{LP}$ the total lift force where F_{L0} is the lift force without the wall effect:

$$F_{L0} = -C_L (\overline{\overline{u_G}} - \overline{\overline{u_L}}) \frac{\partial \overline{\overline{u}}}{\partial y} \quad (22)$$

and F_{LP} is the repulsive force that results form the interaction with the wall; Antal et al (1991) proposed an expression of the wall repulsive force in a laminar flow in the form:

$$F_{LP} = -\frac{2(\overline{\overline{u_G}} - \overline{\overline{u}})^2}{d} \left(\frac{C_{w2}}{y} - C_{w1}\right) \quad (23)$$

We assume that this formulation remains valuable in turbulent bubbly flows, we note y_2^{**} the distance where the wall effect becomes negligible and y_1^{**} the distance where the lift force is zero ($F_{L0} = -F_{LP}$), the lift force in the logarithmic zone can be written in the form:

$$F_L = F_{L0}(1 - f_{LP}(y^{**})) \quad (24)$$

$$f_{LP}(y^{**}) = \frac{y_2^{**} - y^{**}}{y_2^{**} - y_1^{**}} \text{ for } y^{**} < y_2^{**}$$

$$f_{LP}(y^{**}) = 0 \text{ for } y^{**} \geq y_2^{**}$$

IV. RESULTS AND DISCUSSION

The application of the two-fluid model in free turbulent bubbly flows confirms the pertinence of the improvements proposed to ameliorate the predetermination of the turbulence structure in basic bubbly flows: the model predicts correctly the large enhancement of the momentum diffusivity observed in bubbly flow with an important amount of pseudo-turbulence (mixing layer and wake bubbly flows) and reproduces the supplementary amount of the isotropy observed in some sheared bubbly flows (in uniform shear bubbly flows for example). The modification of the redistribution and diffusion modelling was tested in this flow configuration and the value of the coefficient C_R was adjusted to 2/3 (Chahed and Masbernat, 1998a).

The behaviour of the turbulence model in the prediction of near wall bubbly flows experiences e.g. boundary layer (Moursali et al, 1993) and pipe bubbly flows (Serizawa et al, 1986), is also decisive in demonstrating of the pertinence of the turbulence modelling. The model allows reproducing the turbulence structure as it is altered due to the bubble presence in the boundary layer bubbly flows, figure (1) and pipe bubbly flows figure (2).

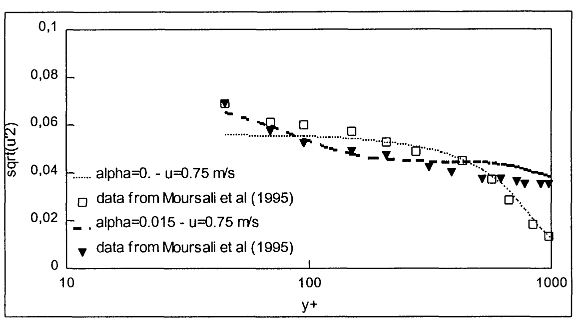

Figure (1): Turbulent intensity in single-phase and bubbly boundary layer flows

These results furnish an explanation to the mechanisms whereby the attenuation of the turbulence in bubbly flows can occurs. The supplementary stretching induced by the bubbles displacements provokes an attenuation of the shear stress, as a result the turbulence production by the mean velocity gradient is reduced and we can note a diminution of the turbulent intensity as observed in some experiences of wall-bounded bubbly flows.

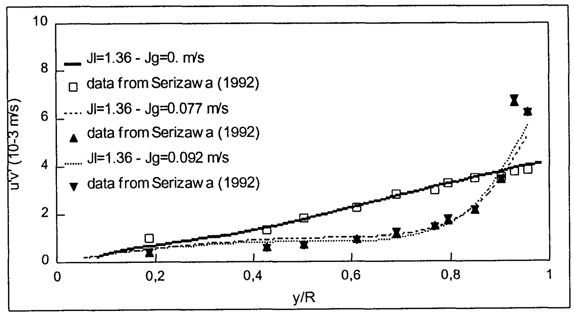

Figure (2): Turbulent shear stress in single-phase and bubbly pipe flows

We specified in Chahed et al (1998a) the important effect of the turbulent correlations issued from the added mass force on the prediction of the void fraction in a bubbly wake. The recent experiences of pipe bubbly flow under micro-gravity conditions (Kamp et al, 1994) provide a remarkable data which allow us to analyse the part played by the turbulent contribution of the interfacial force in the bubbles migration phenomenon. The application of the model to pipe bubbly flow under micro-gravity condition and to upward and downward pipe bubbly flows in normal gravity shows clearly the pertinence of the interfacial force model we propose. An important result of these experiences (figure 1) is to show that, even though the gradient of turbulence in continuous phase is not appreciably modified, the radial void fraction gradient is inverted according as the gravity is active or not (according as the interfacial momentum transfer associated with the average relative velocity is important or not).

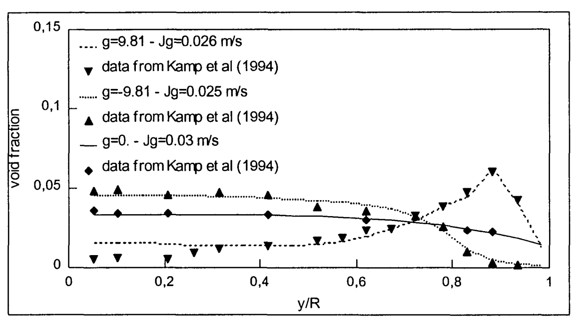

Figure (3): Void fraction distribution in pipe bubbly flow: upward (g=9.81), downward (g=-9.81) and in micro-gravity condition (g=0.)

In micro-gravity condition, the average relative velocity between phases is weak and the action of the continuous phase on the bubbles is reduced to the pressure gradient effect (Tchen force) and to the turbulent contributions of the interfacial force. The pressure gradient effect provokes a bubbles migration toward the wall and can't explain the experimental void fraction profile and the introduction of a supplementary diffusion term proportional to the void fraction gradient can't invert the void fraction profile. In return when the turbulent terms issued from the added mass force are introduced, in our simulation figure (3), the whole action of turbulence is inverted and the phase distribution prediction is in good agreement with the experimental data. This result indicates that the effect of the continuous phase turbulence on the phase distribution includes, beside the pressure gradient action (Tchen force), the turbulent contributions of the interfacial forces. Among these turbulent contributions, the added mass one is particularly expected to have the dominant effect.

As compared to the void fraction profile in micro-gravity condition, the simulations of the void fraction distribution in upward and downward bubbly flows in normal gravity conditions clearly show the effect of the lift force. In the upward flow, the lift force is responsible of the near-wall void fraction peaking while in the downward bubbly flow, the lift force action is inverted and the migration of the bubble toward the centre of the pipe provoked by the global turbulent action is more pronounced than in micro-gravity condition.

The figure (4), shows the simulations of the void fraction profile in boundary bubbly flow with different hypothesis on the interfacial force. These simulations confirm the analysis of the importance of the turbulent contribution of the interfacial force F_t on the void fraction distribution and indicate the effect the lift force F_l and of the wall force F_p on. The adjustment of the coefficients in the expression of the near wall lift force (24) was tested in this flow configuration with different bubbles diameter; it yields $C_L = 0.08$, $y_1^{**} = 1$ and $y_2^{**} = 1.5$.

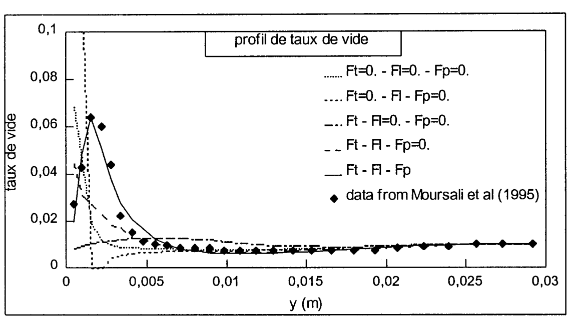

Figure (3): Effect of Turbulence (Ft), Lift (Fl) and Wall (Fp) on the void fraction distribution in boundary layer bubbly flow

These computations allow us to consider that these coefficients could have a somewhat general character. The value of y_2^{**} indicates that, in most of the two-phase flows with millimetric bubbles, the wall force is limited to the logarithmic zone. The value of y_1^{**} suggests that the position of the void fraction peaking is, for the most part, controlled by lift and wall forces: its value corresponds to the void fraction peaking position observed in the experiences of Moursali et al (1995).

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