TRANSITION IN A STRAIGHT PIPE DOWNSTREAM OF A COIL

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ABSTRACT

This investigation is concerned with the (re)transition to turbulence of the flow in a straight pipe downstream of a long helical coil where the flow has previously laminarized. The pipe is tangent to the coil exit and they are both of constant diameter $D = 2R_0$. The coil diameter to curvature ratio is $D/R_C = 1/10.64$ and the coil pitch is small. The Reynolds number of the flow, defined as $Re = DU_b/\nu$, where U_b is the bulk average velocity of the fluid and ν its kinematic viscosity, ranges between 2500 and 12,200. A laser-Doppler velocimeter (LDV) is used to obtain time records, mean and rms values of the streamwise velocity component over the pipe cross-section at various axial locations $2 < x/D < 45$ downstream of the coil-pipe juncture. Limited flow visualization is also performed. The coil exit flow is laminar up to $Re \approx 5200$. That in the straight pipe has transitioned by $x/D = 45$ when $Re = 4600$ and, with increasing Re, the transition location moves upstream while the flow rapidly evolves towards the turbulent state. Analysis of the results suggests that transition to turbulence in the straight pipe may be occurring in *two* ways, through the growth of disturbances induced at the pipe wall, and through an inflectional instability in the core flow.

INTRODUCTION

The flow through a straight pipe connected tangentially to the exit of a helical coil is of considerable engineering and scientific interest because of the stabilizing effects of the coil on the flow. In many industrial and bioengineering applications, turbulent flow through a straight section of pipe passes through a coiled pipe section of the same diameter and then on to another straight section of pipe downstream of the coil. Flows through curved conduits and helical coils have been the subject of considerable research. Early contributions were made by Eustice (1911), Dean (1927, 1928), White (1929) and Taylor (1929). For certain combinations of geometrical and dynamical conditions, characterized by D/R_C and Re (or $De = Re\,(D/R_C)^{1/2}$), it has been shown that the secondary cross-stream motion induced in a helically coiled pipe has the potential for laminarizing a turbulent flow passing through it. To date, the precise manner by which this laminarization occurs remains unexplained.

The helical coil flows investigated by Sreenivasan and Strykowski (1983) [S&S] and Webster and Humphrey (1993) [W&H1] suggest that in a range of Re, exceeding the value required for transition in a straight pipe, a coil acts as a sort of "turbulence filter," tending to suppress high frequency oscillations in the flow passing through it. As Re is increased in this range, but before complete transition to

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turbulence occurs, low frequency oscillations appear in the flow; see W&H1. The calculations of Webster and Humphrey (1997) [W&H2] confirm these oscillations and suggest that they are due to a secondary centrifugal instability, originating near the inner curvature wall where the wall boundary layer turns to join its mirror image, prior to jetting outwards towards the outer curvature wall along the pipe symmetry plane.

Little information is available on the development of the flow in the straight pipe downstream of a coil, especially on transition to turbulence in the presence of the secondary motion induced by the coil. The limited data available, obtained by S&S, suggests that the supercritical laminarized flow in a straight pipe downstream of a coil *does not necessarily retransition to the turbulent state immediately.* Specifically, they observed in a coil with $D/R_C = 1/8.6$ that, as long as $Re < 5200$, approximately, the flow in the straight pipe remains laminar for pipe length to diameter ratios, L/D, as large as 937. Above this value of Re they found that transition takes place spontaneously and is characterized by the appearance of slugs (described as discrete regions of turbulence occupying the entire pipe cross-section; see Wygnanski and Champagne, 1973).

The main objective of this study is to investigate the retransition process in the straight pipe downstream of a helical coil. Measurements of the axial (streamwise) component of velocity of the flow in a range of Reynolds numbers spanning the laminar and turbulent flow regimes at the coil exit are made. For this a laser-Doppler velocimeter (LDV) has been used. Profiles of the mean and rms velocities are obtained as a function of radial location at various axial locations in the pipe. The velocity time records at selected locations also yield information concerning the frequency-energy content of the flow.

EXPERIMENTAL APPARATUS

The experimental set-up is shown schematically in Fig. 1 and is described in detail in Hon (1998). It consists of a pump-driven, gravity-feed, closed-loop water flow system. A constant head is maintained by allowing some overflow from the upper reservoir. The flow enters the straight section consisting of an industrial standard PVC pipe connected to a clear acrylic pipe with a machined polycarbonate joint. The nominal inner diameter of these pipes is 38.1mm and the combined length to diameter ratio (L/D) of the pipes is 80. Machined polycarbonate joints are used to connect the acrylic pipe to the entrance of helical coil and the glass pipe to the coil exit. These joints were machined to match the corresponding ID and OD of the respective tubes they were attached to, thus minimizing any discontinuity there. Flexible industrial Tygon® tubing, with an ID of 38.1 ± 0.4 mm and OD of 47.6 ± 0.8 mm is used as the helical coil. The use of Tygon® has been successfully demonstrated in previous investigations (W&H1, S&S). The coil is formed with a radius of 0.406 m and is supported by a continuous ramp extending from an octagonal frame. The straight pipe section downstream of the coil consists of 2 precision bore borosilicate glass pipes connected by another machined polycarbonate joint for a combined L/D ratio of 56. Before returning to the lower reservoir, the flow is passed through a rotameter and a control valve.

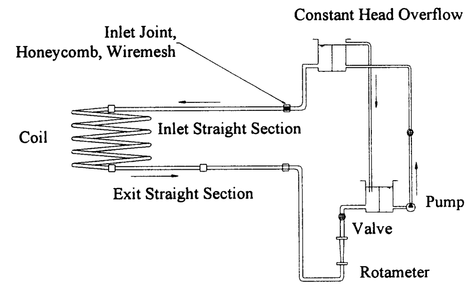

Figure 1. Schematic diagram of the flow setup

The straight pipe exiting the coil is the primary area of interest in this investigation. Borosilicate glass piping was used here for optimal LDV velocity measurements using a 2W Coherent Argon-ion laser. An Aerometrics 2D LDV system was utilized to measure the streamwise velocity component of the flow. The probe volume of the LDV is an elllipsoid with semimajor axes of approximately 0.6 mm and 0.06mm. The water was seeded with 3μm latex particles with a density of 1.05 g/cm^3; these particles were calculated to have negligible settling velocity. A minimum sample of 2000 measurements was taken per measurement location, although most data presented here typically exceed 4000. The mean and standard deviation were calculated from the raw data, and values greater than 2 standard deviations from the mean were rejected. New mean and rms were then recalculated from the reduced data set. Power spectra of the velocity component time records were calculated using the slotted correlation method described by Bell (1986). The integrity of the LDV measurements was determined from two tests carried out in the downstream glass pipe. This was done by reversing the normal direction of the flow so that measurements at $x/D = 45$ for two Reynolds numbers, 980 and 19,000 could be taken. The $Re = 980$ laminar flow results compare well with the analytical parabolic profile and the $Re = 19{,}000$ turbulent flow results are in good agreement with those of Laufer (1954). The uncertainties of the measured data are tabulated in Table 1.

Table 1. Summary of experimental uncertainties

Quantity	Uncertainty(%)
Re, De	±6
U	±4
U_{rms}	±6
r/R_o	±2
$\theta/2\pi$	±0.7
x/D	±1
St	±4

A cylindrical coordinate traversing system was designed to aid with LDV measurements through the curved surface of the pipe. Using this method, the streamwise velocity component at any point in the flowfield could be measured with a simple probe volume location correction such as that presented by Azzola and Humphrey (1984) with the adjustment noted in Webster (1994). The cylindrical coordinate system is defined in Figure 2. The LDV transceiver is mounted on a linear traverse for movement in the radial direction, which is attached to a metal cylinder concentric with the pipe axis. This unit can be rotated around the metal cylinder to allow for measurements corresponding to various polar (θ) angles. The cylinder is supported by a column that slides on a pair of rails that are parallel to the axis of the pipe.

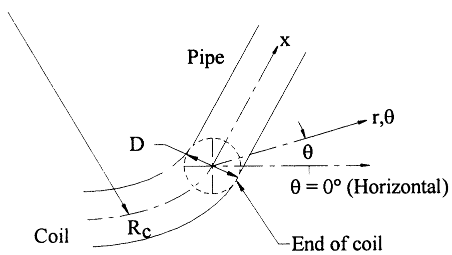

Figure 2. Cylindrical coordinate system for the pipe downstream of the coil.

Flow visualization studies were carried out using neutrally buoyant dye (methylene blue) injected at the outlet of the coil. The dye could be injected through any of sixteen 0.08mm diameter ports, spaced equally at the connection joint between the coil and the glass pipe. Additional flow visualization was done by illuminating the flow seeded with mother of pearl platelets with a laser sheet.

RESULTS AND DISCUSSION

Profiles of the axial (streamwise) velocity component mean and rms have been measured along the θ = 0° - 180° horizontal pipe diameter at *x/D* = 2, 4, 12, 20, 32.5 and 45 for *Re* = 2500, 5200, 6300, 7700, and 10,200 and at *x/D* = 45 for *Re* = 4000 and 4600. An additional profile was taken at *x/D* = 2 for *Re* = 4000. Similar profiles have been measured along pipe diameters starting at θ = 60°, 30°, 0°, -30°, -60°, -90° and –105° for *x/D* = 2 and 12 with *Re* = 5200, and 10,200. (All the data have been obtained by initiating the radial traverse at the θ location noted.) The mean and rms profiles presented here were measured along the θ = 0^0-180^0 pipe diameter. Limited mean and rms measurements were also made at the end of the upstream tangent. They indicate that the flow is laminar for *Re* = 2500 and fully turbulent for *Re* ≥ 4000. These and other related results are presented in Hon (1998) and only selected findings are discussed here.

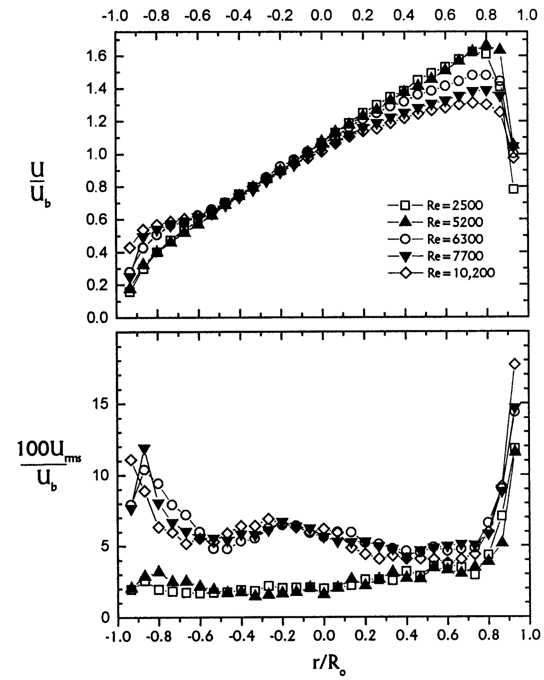

Figure 3. Mean and rms velocity profiles at x/D = 2.

The characteristics of the flow at the entrance to the downstream tangent are determined by the Reynolds number. Figure 3 shows the mean and rms profiles measured at *x/D* = 2 for different values of *Re*. The mean velocity profiles show the centrifugal force effects induced by coil curvature on the flow, the maximum velocity being displaced towards positive *r/Ro*, corresponding with the outer curvature wall of the coil. The normalized rms profiles indicate that the data tends to fall into two categories, one for which the coil exit flow is fully turbulent and another for which it is laminar or laminarized. For *Re* ≥ 6300, the flow in the upstream tangent is turbulent and the flow in the coil is fully turbulent. In the *Re* range between 4000 and approximately 5200, the flow in the upstream tangent is turbulent but that at the coil exit plane is laminarized. In this range of Reynolds number, the stabilizing effects of the coil flow damp the turbulence in the inlet flow either completely or quite substantially, depending on the Reynolds number. For *Re* = 2500, the flow is laminar everywhere.

Figure 4 shows the measured mean and rms profiles at *x/D* = 45, while Figure 5 shows the *x/D* = 45 data normalized for comparison with the fully developed flow results of Laufer (1954). By *x/D* = 45 when *Re* ≥ 4600, the flow is already turbulent, the degree of approximation to the fully developed condition depending on *Re*. In contrast, for *Re* = 2500, the rms velocity remains relatively low while the mean has partly evolved towards a parabolic (laminar flow) profile. We will focus on the *Re* = 5200 case as it is the highest *Re* case for which the coil exit flow is laminarized. At this Reynolds number, although the coil exit flow is laminar, oscillations with a predominant frequency corresponding to a St = 0.25 are present in the inner half of the pipe flow. These oscillations were observed by WH1 in their experiments and are discussed further in WH2.

x/D for different values of Re. The figure shows that for Re = 2500 the flow in the pipe remains laminar. Beyond this value, the flow ultimately transitions to turbulence. This is a gradual process for Re = 5200 and, in all cases, there is an overshoot in the value of the rms with respect to the asymptotic value at $x/D \geq 45$.

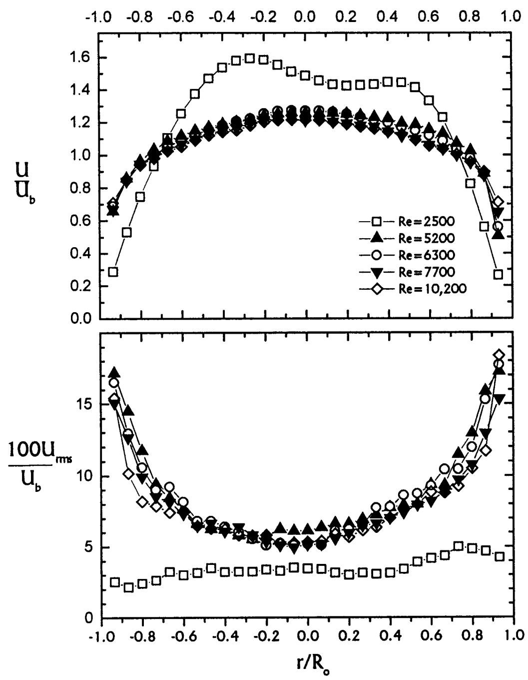

Figure 4. Mean and rms velocity profiles at x/D = 45 .

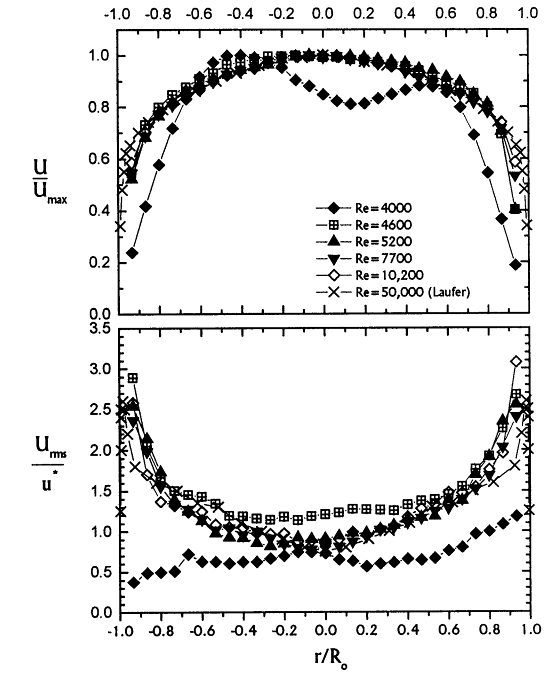

Figure 5. Mean and rms velocity profiles at x/D = 45 compared to Laufer's fully developed turbulent flow results.

Figure 6 illustrates the axial development of the flow measured along the horizontal pipe diameter for Re = 5200. (Note that while this diameter is contained in the horizontal symmetry plane of the straight pipe, because of the slight coil pitch the flow in the pipe is not completely symmetrical about it.) The mean velocity profiles from x/D = 4 to 20 display inflection points which result from the coil-induced secondary motion. This motion convects a significant amount of fluid with a high axial component of momentum around the periphery of the straight pipe and traps a region of low speed fluid in the core region of the pipe. The rms profiles increase markedly between x/D = 4 and 12 and by x/D = 32.5, turbulent diffusion of momentum has significantly altered the mean velocity profile. Noteworthy are the overshoots in the rms, particularly at x/D = 4 and 12 which correspond closely with the point of inflection in the mean velocity profiles. The distributions of the rms suggest that the flow is experiencing a spatially complex transition to turbulence in the pipe.

Values of the rms velocity, averaged along the pipe horizontal diameter, are plotted in Figure 7 as a function of

Power spectra of the velocity signals were obtained from long time records taken along the pipe horizontal diameter for selected values of x/D and Re. For Re = 5200, spectra were measured from x/D = 2 to 45. From these spectra, plots of the predominant frequency and its harmonics along with their associated energy content were obtained. Figure 8 gives the predominant St and associated energy as a function of radial position for x/D = 2. At this axial location, the flow contains a predominant frequency corresponding to a $St \approx$ 0.25 for $r/R_0 < 0$, which is in good agreement with the results of W&H1 in their coil. The oscillation energy is largest near the inner wall and there is a very limited presence of the first harmonic. The outer region is devoid of these oscillations induced in the coil at x/D = 2. W&H2 attributed this predominant frequency to a travelling wave originating from a secondary centrifugal instability near the inner curvature wall of the coil. The corresponding plots for x/D = 12 are shown in Figure 9. At this axial location, the flow contains oscillations over a large fraction of the pipe diameter with a $St \approx 0.25$. The oscillation energy is largest near the pipe center and for negative values of r/Ro, corresponding to the inner curvature wall of the coil. The spectral data show that the oscillations spread radially across the pipe from the inner wall with increasing x until $x/D = 20$ where almost the entire pipe contains the oscillations. These oscillations also grow in amplitude until $x/D \approx 20$, where significant higher harmonics are present. At $x/D = 32.5$, no dominant frequencies are present in the flow and the spectra are characteristic of turbulent flow. At this location, the spectra at $r/R_0 = \pm\ 0.5$ are not identical, indicating that the flow is turbulent, but not yet fully developed.

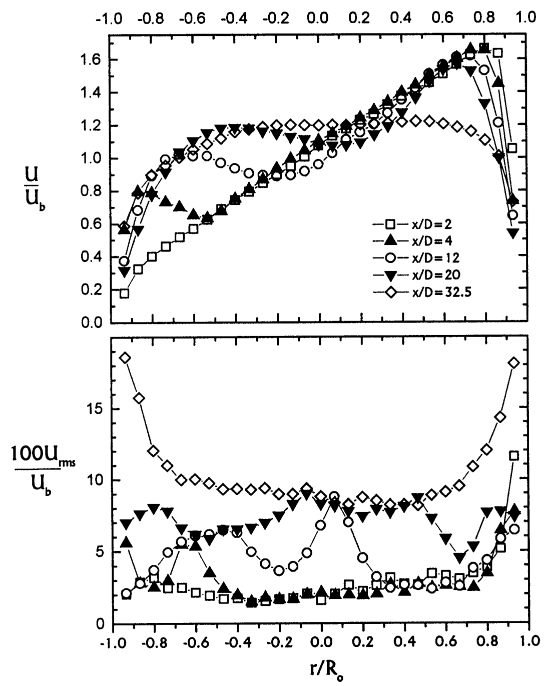

Figure 6. Axial development of mean and rms velocity profiles for Re = 5200.

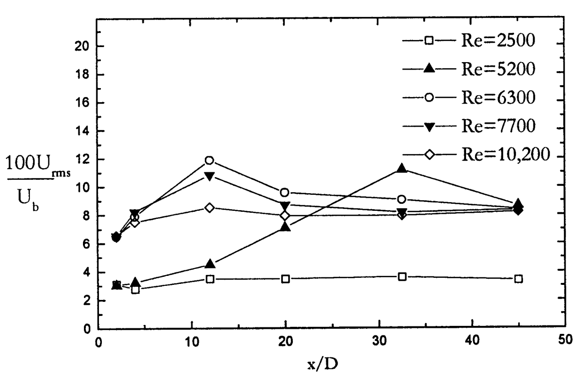

Figure 7. Rms averaged over the horizontal pipe diameter ($\theta = 0^0$) at various x/D.

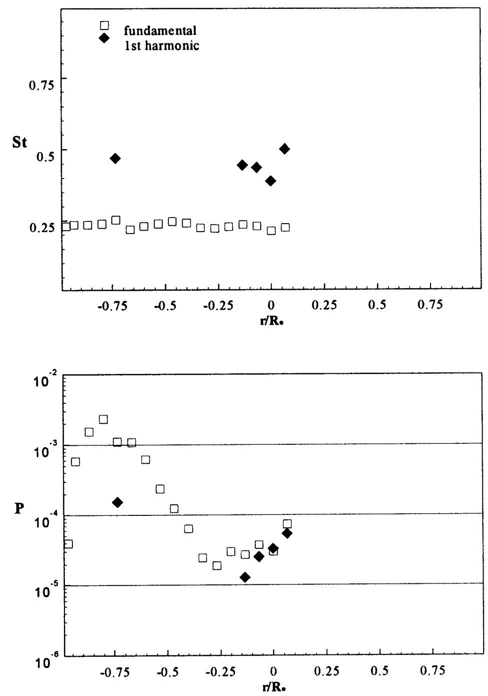

Figure 8. Strouhal number and energy content of the axial velocity fluctuations at $x/D = 2$ and $Re = 5200$.

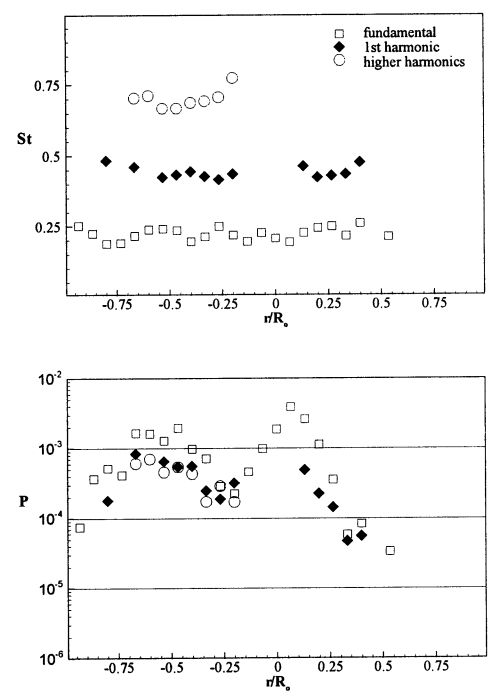

Figure 9. Strouhal number and energy content of the axial velocity fluctuations at $x/D = 12$ and $Re = 5200$.

CONCLUSIONS

The physical mechanisms by which a fluid undergoes transition to its final turbulent state are expected to depend significantly on the initial conditions of the motion in the laminar(ized) state. For the $Re = 5200$ case, the inlet flow to the downstream tangent is markedly asymmetrical with a significant cross-stream component of motion. The transition process for $Re = 5200$ appears to begin at $x/D = 2$ in a region of the pipe nearer the inner wall where a coil-generated predominant frequency with $St \approx 0.25$ exists. The spectral data show that the region containing the oscillations grows spatially across the pipe and the amplitude of the predominant frequency increases with increasing x/D. The oscillatory region spreads across the entire pipe by $x/D \approx 20$, where significant higher harmonics are also present. The flow becomes fully turbulent by $x/D = 32.5$. Simultaneously, the coil-induced cross-stream motion generates axial velocity profiles that contain inflectional points out to $x/D \approx 20$. Peaks in the radial rms distributions occur approximately at the radial locations of the inflectional points. At $x/D = 4$, the inflection point is located in the region of $r/R_0 \approx -0.6$ and $\theta = 0^0$. The location of the inflection point moves radially towards the pipe center with increasing x/D. Rayleigh's criterion for (inviscid) instability of a two-dimensional shear flow requires the existence of inflection points. However, the present flow field is far more complex than a two-dimensional parallel shear flow and the additional necessary condition (see White, 1991) that the vorticity component normal to these profiles maximize at the points of inflection, is not met.

The results suggest that for $Re = 5200$, it is possible that transition to turbulence may be occurring two ways, through the growth of disturbances induced near the inner pipe wall and through an inflectional instability in the core flow. Further, the results show that transition to turbulence in a straight pipe tangent to the exit of a helical coil occurs at lower Re than previously reported. At $x/D = 45$, transition occurs for $Re \geq 4600$ for the laminarized pipe flow exiting from the coil.

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