DEVELOPMENT OF EXPERIMENT/SIMULATION INTERFACES FOR HYBRID TURBULENT RESULTS ANALYSIS *VIA* THE USE OF DNS

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ABSTRACT

In this numerical study, a possible interface between experiment and simulation is developed. The main goal is to be able to interface experimental time histories, measured at some few locations, with the inlet section of a time varying numerical simulation of a spatially developing flow. The study is performed through two-dimensional DNS of a mixing layer by defining a virtual interface within the computational domain. First, necessary criteria to insure "realistic" inflow conditions for numerical simulations are analyzed. It is then shown that even if the Reynolds stresses and spectral distributions are properly taken into account, a correct representation of the space-time coherence of the flow organization is essential to obtain a good interface. Using the Linear Stochastic Estimation (LSE) of the whole inlet velocities from only three spatial monitoring locations, "realistic" downstream velocity fields can be generated. This realism is found to be valuable as well for turbulent statistics as for instantaneous snapshots of vorticity of the flow field. Two main applications of the present interface are then possible: experimental unsteady conditions can be used to drive a DNS, while the numerical simulation can be helpful as a tool for dynamical signal processing analysis of experimental data.

INTRODUCTION

Taking into account the complementary nature of experiments and computations (Moin and Mahesh, 1998) can lead to a very interesting association from which hybrid results keeping the main advantages of both approaches could be obtained. The main problem underlying this association stays in determining an optimal procedure that makes it possible. The objective of the present work is to test various possibilities that can lead to carry out an interface between experiments and computations. Such an interface consists in using experimental data as inflow conditions for non-stationary computations such as Direct Numerical Simulation (DNS) or Large Eddy Simulation (LES). This real/virtual interface (see Figure 1) has to be optimized in order to avoid degradation of incoming flow characteristics.

To obtain experimental spatio-temporal information, two major methods are usually available: quantitative visualizations (*PIV-HPIV*) or simultaneous measurements by distributed sensors (for example *hot-wires rakes*). The first solution provides a quite satisfactory spatial resolution, but the associated temporal resolution remains quite insufficient for an accurate description of the non-stationary character of most flow configuration. Conversely, the high frequency bandwidth of the second method provides a satisfactory representation of the temporal evolution of the flow. In this second case however, simultaneous measurements of the three velocity components are only possible at a limited number of points. Thus, with such an experimental arrangement, only a limited knowledge of the spatial organization of the flow is effectively available. Consequently, it is required to develop an optimal interpolation/extrapolation procedure permitting to estimate, from a small number of velocity measurements, a complete velocity field at each numerical grid point located at the inflow boundary of the computational domain. To summarize, the experiment/simulation interface needs to address the following three steps:

- *velocity measurements* by a limited number of probes. The location of these probes has to be optimized in order to gain maximum significant information about the spatial organization of the flow;
- *reconstruction of velocity data* in the measurement section at a given set of grid points, *via* interpolation/extrapolation procedures;
- *simulation of the downstream development* of the flow, using the resulting reconstructed velocity data as inflow conditions.

It should be emphasized that such an experiment/simulation association can be considered as a procedure for generating "realistic inflow conditions" for a numerical simulation, but also as a procedure for a "dynamical data analysis", where DNS/LES computations

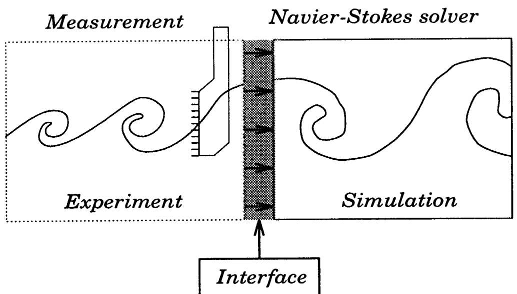

Figure 1. Schematic representation of an experiment-simulation interface for the typical case of a spatially developing mixing layer.

are used to improve experimental data by restoring the part of the information that has been deteriorated during the first two steps.

The topic of this paper is to evaluate the effects of the last two steps. For this purpose, a preliminary knowledge of the complete flow would be necessary to accurately quantify their effects. Experimentally, this condition is nowadays not affordable even if experiments with a large amount of hot-wires have been already performed (Delville *et al.*, 1999 (48 hot-wires), Ewing and Citriniti, 1997 (138 hot-wires)). That's why we chose to use a DNS to characterize the effects of each step. From this DNS, a virtual interface (within the computational domain) is defined on which various possible approaches are applied. In other words, the DNS data will replace the measurement part of the chart given on figure 1.

After a description of the numerical method used, the methodology is presented. The velocity reconstruction step and the simulation step are then analyzed in terms of their ability to provide satisfactory instantaneous vorticity fields and statistical quantities as well.

NUMERICAL METHOD

A two-dimensional spatially developing mixing layer between two streams (velocities U_1 and U_2) is considered. The difference and ratio of velocities are respectively noted $\Delta U = U_1 - U_2$ and $\lambda = (U_1-U_2)/(U_1+U_2)$, with $U_1 > U_2$ and $\lambda = 0.4$ for the present study. The computational domain is chosen large enough so that two pairings of Kelvin-Helmholtz vortices can occur. The domain size is $(L_x, L_y) = (280\ \delta_{\omega_i}, 56\ \delta_{\omega_i})$, where δ_{ω_i} denotes the vorticity thickness of the velocity profile at $x = 0$. The Reynolds number based on δ_{ω_i} and ΔU is $Re = 200$.

The numerical code solves the incompressible two-dimensional Navier-Stokes equations in a computational domain discretized on a regular and non-staggered grid of $(n_x \times n_y) = (1001 \times 201)$ points. Sixth order compact centered differences schemes are used (Lele, 1992) to evaluate all spatial derivatives, except near the in- and outflow boundaries where single sided schemes are employed for x-derivatives calculation. Equations are integrated in time using a third order Adams-Bashforth scheme (AB3). In spite of its stability properties and storage requirement less favourable than Runge-Kutta schemes (RK3), the AB3 scheme does not require processing sub-time-step for inflow condition generation. The incompressibility condition is ensured with a fractional step method up to machine accuracy.

Free slip boundary conditions are applied to $y = \pm L_y/2$, allowing to preserve the same numerical scheme for y-derivatives calculation in the whole computational domain (symmetry and anti-symmetry conditions). Outflow boundary conditions at $x = L_x$ are determined through the resolution of a simplified convective equation.

For inflow conditions application, velocity components (u, v) are prescribed at each grid point without any particular numerical treatment. The generation of the inflow data is deduced either from a previous simulation or from a random fluctuating velocity field of small amplitude superimposed on a basic tangent hyperbolic profile (only for the first simulation).

METHODOLOGY

The method consists first in performing a DNS of a spatially developing plane mixing layer in a large computational domain, this first simulation being considered as the reference case. Note that present DNS results are in a good agreement with previous studies on this subject (Druault *et al.*, 1998). During the calculation, streamwise and transverse velocity components (u, v) are stored from a deliberately limited number of grid points (considered as "numerical probes") among the 201 available ones at a given streamwise location $(x_0 = 180\delta_{\omega_i})$. Then, data obtained from these numerical probes are used to reconstruct (by interpolating or extrapolating in space) the entire velocity field at each grid point located at $x = x_0$ by using various procedures (second step). Finally, a second DNS is performed on a truncated computational domain (in its upstream part) using reconstructed velocity field as inflow condition (third step).

With such purely numerical approach, it is possible to get a precise description of the total degradation associated with the interface by comparison with reference DNS results.

LOCAL RECONSTRUCTION

One of the objectives is to keep the number of reference points as small as possible (Druault *et al.* 1999a). Indeed, if 3D configuration is addressed, it would be necessary to consider both the mean shear direction and spanwise one. Because the number of probes requirement is *a priori* rather severe in the spanwise direction, we take into account only 3 reference y-locations points. For the determination of optimal positions, recommendations given in previous experimental works are followed (Cole *et al.*, 1992 and Vincendeau, 1995). More precisely, locations of reference points correspond to $\pm\delta_{\omega_0}/2$ and to the mixing layer y-center (δ_{ω_0} is the vorticity thickness at $x = x_0$). From these reference data, we have previously shown that reconstruction methods based on "physical" concepts (two-points correlation tensor) are more suitable than traditional procedures based on mathematical interpolations (linear or splines, see Druault *et al.*, 1998 for details). Thus, in this paper, the reconstruction using the Linear Stochastic Estimate (LSE, Adrian, 1995) is used. Note that the complementary method (Bonnet *et al.*, 1994) (LSE+ Proper Orthogonal Decomposition) could also be useful in the context of high Reynolds number flows in a fully turbulent state. In the present situation, such an approach does not involve notable modifications in comparison with LSE procedure (Druault *et al.*, 1998, Druault *et al.* 1999a).

Linear Stochastic Estimation

The Stochastic Estimation has been initially introduced by Adrian in order to provide a "conditional estimate" of the large scale structures present in turbulent flows (Adrian, 1995). This estimation uses the "conditional information" specified at one or more y_{iref} locations in conjunction with the statistical properties (two-points correlation tensor $R_{ij}(y', y_{iref}) = \langle u_i(y')u_j(y_{iref})\rangle$) to estimate the information at surrounding locations y'. Tung et Adrian (1980) have shown that the influence of high order terms on the estimate is not significant, justifying the use of a simple Linear Stochastic Estimation (LSE). More precisely, at a given location y', an estimate of the velocity component $\hat{u}_i$, is expressed as a function of u_j at locations y_{iref}:

$$\hat{u}_i(y') = A_{ij}^{iref} u_j(y_{iref}) \quad (1)$$

The coefficients A_{ij}^{iref} are determined by a least mean square procedure which leads to the resolution of the following system:

$$A_{ij}^{iref} \langle u_j(y_{iref})u_n(y_{jref})\rangle = \left\langle u_i(y')u_n(y_{jref})\right\rangle \quad (2)$$

The determination of the coefficients A_{ij} is based on conventional two-points correlation tensor $R_{ij}(y', y_{iref})$, and coefficients A_{ij} are time independent. Note that this method allows to estimate velocity field only at y' where the matrix A_{ij} has been determined i.e. where correlation tensor $R_{ij}(y', y_{iref})$ is known. In this study, the entire reference DNS data are used to compute these coefficients. For real experiments, some interpolation/extrapolation procedures have to be developed in order to estimate the full two-point spatial correlation tensor on a finer and broader mesh than practically possible (Druault *et al.*, 1999b).

Local velocity field reconstruction at $(x = x_0)$

From the knowledge of the three reference points data, the instantaneous velocity field (u, v) is reconstructed by LSE for each grid point at x_0. As Druault *et al.* (1998) previously noted, the fluctuating kinetic energy reconstruction is in good agreement with the exact one. Here, in order to illustrate this reconstruction, we chose to represent temporal sequences of the instantaneous vorticity field. This last quantity is obtained by using Taylor hypothesis: $\frac{\partial}{\partial x} = -\frac{1}{U_c}\frac{\partial}{\partial t}$. Note that for the present flow, the validity of this hypothesis has been checked (Druault *et al.*, 1999a). Figure 2 compares at $x = x_0$ the "exact" temporal sequence of vorticity field (deduced from Taylor hypothesis) with the one reconstructed by LSE from the 3 reference points. On this figure, the large scale structures behavior is globally well reproduced, while details of the vorticity within the braids are deteriorated. This observation underlines the difficulty to obtain experimentally the spatial structure of the vorticity field with precision (i.e. including the small scales contribution).

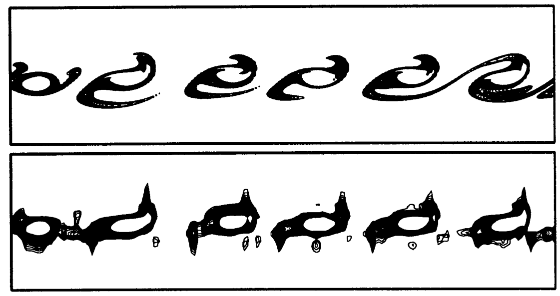

Figure 2. Temporal sequences of instantaneous spanwise vorticity at $x = x_0$, $\omega_z = -\frac{1}{U_c}\frac{\partial v}{\partial t} - \frac{\partial u}{\partial y}$ (using Taylor hypothesis, $U_c = (U_a + U_b)/2$: convection velocity): exact reference one (top) and reconstructed one with LSE procedure from 3 y-reference locations (bottom).

INFLOW SELECTED CRITERIA

In the context of inflow condition generation problem for numerical simulations, two main questions can be addressed:

- What are the significant criteria to be checked to ensure realistic inflow conditions?
- What is the ability of a numerical simulation to improve the realistic character of the flow during its spatial development?

In order to address these questions, we compare five cases simply differing by their inflow conditions. The computation domain now considered (L'_x, L_y) is the reference one, truncated in its upstream part, with $L'_x = L_x - x_0$. Each case is referred as follows

- case (a): no degradation (exact, reference case);
- case (b): y and time phase jittering of the reference data at $x = x_0$ (white noise) preserving the Reynolds stress tensor;

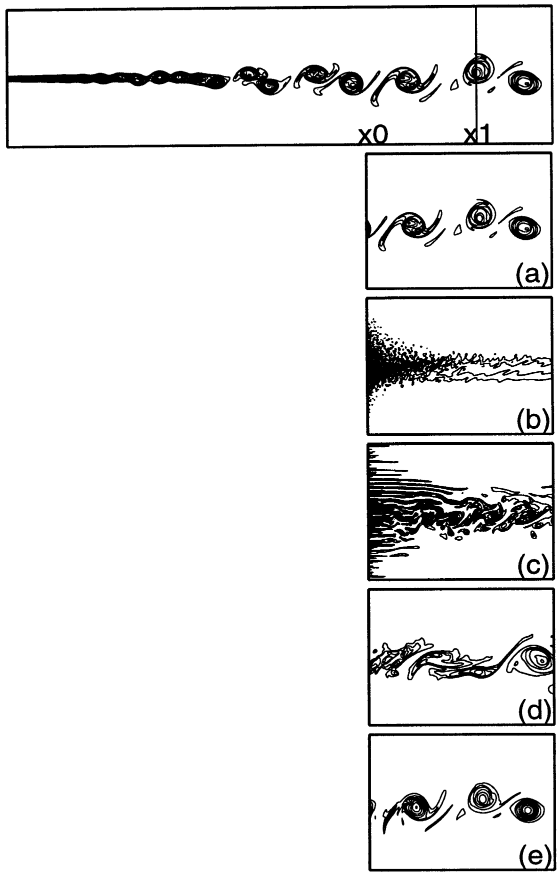

Figure 3. Spanwise vorticity contours. Reference simulation ($L_x = 280\delta_{\omega_i}$) (top). Below: Truncated simulations ($L'_x = 100\delta_{\omega_i}$) using inflow conditions (a), (b), (c), (d) and (e) detailed in the text.

- case (c): same as case (b) but preserving the temporal spectrum of each component of velocity (Lee *et al.*, 1992);
- case (d): same as case (c) but without phase jittering in y-direction;
- case (e): instantaneous velocity field reconstructed by using LSE procedure from the knowledge of stored data at only three reference points.

A comparative analysis of these inflow conditions is performed by studying the instantaneous vorticity field (Figure 3) and the fluctuating kinetic energy obtained at $x_1 = 240\delta_{\omega_i}$ (Figure 4).

The perfect agreement between case (a) and the field issued from the (initial) full computation means that single sided schemes used for x-derivatives calculation at $x = x_0$ for case (a) have no noticeable effect on the result. Conversely, case (b) results appear unacceptable, leading to a strong underestimation of the fluctuating kinetic energy (loss of 99 %) in the section $x = x_1$. The preservation of the spatial distribution of the Reynolds

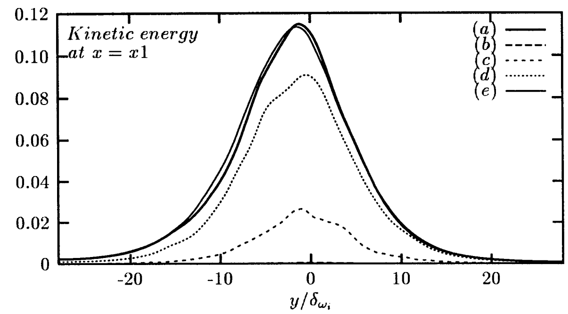

Figure 4. Fluctuating kinetic energy at $x = x_1$ obtained from previous truncated DNS simulations (a), (b), (c), (d) and (e).

tensor alone seems to be very insufficient to obtain a realistic dynamics of the simulated flow. The case (c) confirms this previous result, even if the conservation of the temporal spectrum of each velocity component permits to limit the fall of the fluctuating kinetic energy which rises to approximately 80 % (Figure 4). The analysis of case (d) results shows a clear improvement of the realistic character of simulation. We can firstly conclude that the correct representation of temporal and spatial energy spectrum is a favourable condition for numerical simulations realism. Nevertheless, note that the time phase jittering applied to inflow conditions for case (d) means that these conditions do not correspond to a solution of Navier-Stokes equations (absence of coherent vortices). In spite of this lack of realism, it is interesting to observe that the simulation acts as a dynamical reconstruction of condition (d) allowing to lead downstream to the formation of "realistic" vortices. This role is also observed with case (e) results for which a better agreement is observed between estimated instantaneous vorticity field and the original one. It can be noticed that numerical simulation in this case allows to appreciably improve the realism of the flow during its downstream development. Conversely, the faithful preservation of the Reynolds tensor and spectral characteristics of the flow at inlet (without any coherent character) turns out to be too disadvantageous to observe a similar improvement.

Note that for the implementation of experiment/simulation interface, only inflow conditions such as (b), (c) and (e) are possible. As a conclusion, the realization of an experiment/simulation interface can be favourably performed from a limited knowledge of experimental data if the spatio-temporal coherence of data is preserved (case (e)) rather than only their spatio-temporal (statistical) energy distribution.

DYNAMICAL SIGNAL PROCESSING

In this part, we will compare at a given section ($x = x_1$) the temporal sequences of the exact vorticity field with its estimation accessible experimentally.

As we previously noted, the estimation of vorticity field can be obtained experimentally only from a limited number of y-positions. Thus, in the same way that performed for the previous section $x = x_0$, we can reconstruct the time-dependent velocity field in the section $x = x_1$ by using LSE procedure. Only 3 reference points are still used at $y = -\delta_{\omega_1}/2, 0, \delta_{\omega_1}/2$ where δ_{ω_1} is the vorticity thickness at $x = x_1$. From this reconstruction, the temporal sequence of vorticity field is deduced (figure 5-3) from Taylor hypothesis. According to the previous results obtained in $x = x_0$, reconstructed temporal sequence of vorticity field appreciably differs from the "exact" one (see figures 5-1/5-3), the vorticity details in braids being strongly deteriorated.

The solution suggested in this study consists in performing an experiment/simulation interface allowing to restore quickly (in terms of eddy turn over-time) most of the realistic details of the vorticity field during the simulation step. The result obtained with such an approach is shown on the figure 5-2, where the same temporal sequence of the vorticity deduced from calculation (e) is presented. It can be noticed the clear improvement of the vorticity data obtained. Such an restitution suggests that numerical simulation could be used as a tool of "dynamical data analysis", allowing to improve the measurements quality. Note that many realistic details of vorticity data are restored rather quickly (here, in a development time $T_r \simeq (x_1 - x_0)/U_c$). Indeed, the hybrid nature of such a procedure (in a real experiment/simulation interface) let us foresee many possibilities: for instance, analysis of the flow spatial structure, access to pressure-velocity correlations, applications to turbulent flows control, *etc.*.

CONCLUSION AND PROSPECT

In this work, the possibilities and the interest of the implementation of an experiment/simulation interface have been studied. In order to well determine the potential and the limitations of such an approach, the present study was firstly limited to a purely numerical approach for which we have virtually defined an interface within the computational domain. The major conclusions are the following ones

- to perform an experiment/simulation interface, it is more useful to preserve the spatio-temporal coherence of inflow data rather than exact space and spectral energy distributions associated to their fluctuations;
- the use of the reconstruction method based on Linear Stochastic Estimation seems to be efficient for inflow condition generation;
- the simulation step tends to dynamically finalize the data reconstruction;
- the implementation of an experiment/simulation interface can be conceived like a procedure of dynamical signal processing, allowing to generate a result of hybrid nature.

Figure 5. Temporal sequences of instantaneous spanwise vorticity at $x = x_1$ from truncated 2D-DNS. (1) and (2): sequences respectively obtained with case (a) and (e) $\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$. (3): reconstructed temporal sequence with LSE procedure from 3 y-reference locations in section $x = x_1$ and using Taylor hypothesis $\omega_z^e = -\frac{1}{U_c}\frac{\partial v}{\partial t} - \frac{\partial u}{\partial y}$.

Naturally, the conclusions brought by this work must be confirmed in a more realistic three-dimensional context of turbulence. Very recently, a similar procedure has been done in a 3D DNS with the following parameters: $Re = 200, (L_x, L_y, L_z) = (100\delta_{\omega_i}, 50\delta_{\omega_i}, 25\delta_{\omega_i}), (n_x \times n_y \times n_z) = (301 \times 97 \times 48)$, where δ_{ω_i} denotes the vorticity thickness at inlet. A same numerical method is used for time integration and spatial derivatives, and a periodic condition is used for the spanwise direction. The same methodology described above is then followed: we performed firstly a reference simulation in a large computational domain ($L_x = 100\delta_{\omega_i}$). During this calculation, instantaneous velocity data are stored at a given streamwise section ($x_2 = 50\delta_{\omega_i}$). Then, for each spanwise location, 3 y-locations (respectively positioned at $y = \pm\delta_{\omega_2}/2$ and $y = 0$), are considered as reference numerical probes (δ_{ω_2} corresponds to the vorticity thickness at x_2). After that, for each z-position considered, the LSE procedure is used to reconstruct the instantaneous velocity field at each grid y-point located in this streamwise section. Finally, a second 3D DNS is performed on a truncated computational domain ($L'_x = L_x - x_2$) using the reconstructed velocity data as inflow condition. The obtained results are then compared with the reference ones. Figure 6 presents a vorticity modulus isosurface for such truncated 3D DNS. A quite satisfactory agreement is observed between both cases showing the efficiency of stochastic estimation in a 3D configuration.

Nowadays, instead of using the knowledge of velocity field at 3 y-locations for every z-grid points, our goal is to take into account only a reduced number of z reference locations in order to limit the number of probes requirement for the real experimental/numerical interface. The first tests of local reconstruction show the need to develop a specific reconstruction method for the homogeneous z-direction. Indeed, although Linear Stochastic Estimation is well suited for inhomogeneous y-direction, it is not well designed for homogeneous direction such as the z-one. The major problem concerns the adaptation of measurement data to periodic boundary conditions and homogeneous character in z-direction. In spite of these additional difficulties for a three-dimensional approach, it is anticipated that vortex stretching mechanisms would accelerate the restoring phase of the missing part of the motion, then compensating the lack of spatial resolution of three-dimensional measurement data.

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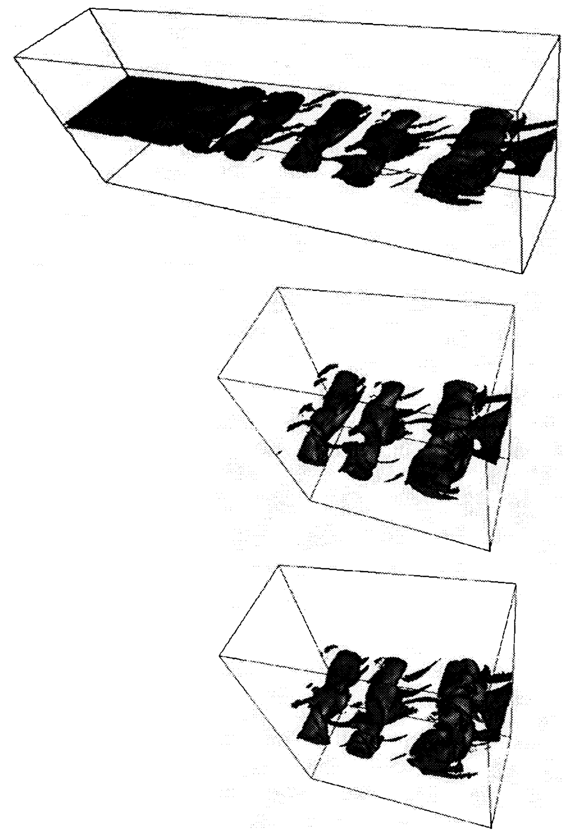

Figure 6. Spanwise vorticity surface. Reference simulation ($L_x = 100\delta_{\omega_i}$) (top). Below: Truncated simulations ($L'_x = 50\delta_{\omega_i}$) using exact inflow conditions (center) and velocity field reconstructed with LSE procedure from the knowledge of stored data at 3 y-points in each z-location (bottom).

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