THE MEASUREMENT OF LONGITUDINAL AND TRANSVERSE STRUCTURE FUNCTIONS IN A TURBULENT ROUND JET

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ABSTRACT

The behaviour of longitudinal and transverse structure functions in the far field of a turbulent round jet is investigated experimentally. Attention is focused on the scaling exponents of structure functions within the inertial range. Different initial conditions represented by different turbulent intensities at the jet nozzle are considered. The difference between the scaling exponents of longitudinal and transverse structure functions is found to be almost independent of the initial conditions. The majority of this difference depends on the anisotropy of the flow field and the difference also decreases with increasing Reynolds number.

INTRODUCTION

It is now a very well know result that the moments of the velocity difference between two points (the velocity structure functions, hereafter SF) in a high Reynolds turbulent flow scale as a power of the distance between the points (Frisch, 1995). Following the Kolmogorov (1941) local similarity hypothesis (hereafter K41), the scaling exponent increases linearly with the moment of the SF. However, departures from the K41 scaling have been extensively reported especially for longitudinal structure functions (LSF), that is for the statistics of the differences of velocity components along the direction of separation (see Antonia and Sreenivasan (1996) for a review). Less attention has been given to transverse structure functions (TSF) or differences of velocity components orthogonal to the separation distance. However, especially in anisotropic flows, a complete investigation of intermittency must include information from both longitudinal and transverse directions (also in view of the determination of the scales of coherent structures in the two directions).

For locally isotropic turbulence of an incompressible flow the fundamental relation between second-order LSF and TSF ($\overline{\Delta_r u_{L,T}^{\;2}}$) is given by (Monin and Yaglom, 1975):

$$\overline{\Delta_r u^{*}_{T}{}^{2}} = \left(1 + \frac{r}{2}\frac{\partial}{\partial r}\right)\overline{\Delta_r u^{*}_{L}{}^{2}} \qquad (1)$$

where $r^{*}=r/\eta$ is the normalized separation distance (η is the Komogorov length scale $=(\nu^3/\varepsilon)^{1/4}$) and $u^{*}=u/v_K$ is the normalized velocity component along the direction r (v_K is the Kolmogorov velocity = $(\nu/\varepsilon)^{1/4}$). Assuming scaling properties for both LSF and TSF within the inertial range ($1 \ll r^{*} \ll Re^{3/4}$, where Re is the Reynolds number based on integral scale), i.e.

$$\overline{\Delta_r u^{*}_{L,T}{}^{p}} \approx C_p^{L,T} r^{*\varsigma_p^{L,T}},$$

the use of (1) implies that the scaling exponents are related by

$$\varsigma_2^T = \varsigma_2^L + \log_{r^*}\left(\frac{C_2^L}{C_2^T}\left(1 + \frac{\varsigma_2^L}{2}\right)\right) \qquad (2)$$

where C_2^L and C_2^T are the constants involved in the second-order scaling law. When the same velocity fluctuations, for example u, is used to form both LSF and TSF, one would expect that the difference between ξ^L and

ζ^T should be only small. However, the direction of the transverse separation is likely to be important. A relation similar to (1) was derived for the fourth-order SF, using an empirical approach (Antonia *et al.*, 1997)

$$\overline{\Delta_r u^*{}_T{}^4} = \left(1+\frac{r}{4}\frac{\partial}{\partial r}\right)\left(1+\frac{r}{4}\frac{\partial}{\partial r}\right)\overline{\Delta_r u^*{}_L{}^4}. \quad (3)$$

This leads to the following relation between fourth-order scaling exponents

$$\varsigma_4^T = \varsigma_4^L + \log_{r^*}\left(\frac{C_4^L}{C_4^T}\left(1+\frac{\varsigma_4^L}{2}+\left(\frac{\varsigma_4^L}{4}\right)^2\right)\right), \quad (4)$$

where C_4^L and C_4^T are the constants involved in the fourth-order scaling law. Equations (1) and (3) can be used to assess which scales conform with isotropy and allow isotropic values of the transverse exponents to be estimated from measured longitudinal exponents. These isotropic values can be compared with the true measured transverse exponents.

With no intermittency, K41 at large Reynolds numbers yielded $\zeta_p^L = p/3$. Using (2) and (4), $C_2^T/C_2^L = 4/3$ and $C_4^T/C_4^L = (4/3)^2$, so that

$$\zeta_{2,4}^T = \zeta_{2,4}^L. \quad (5)$$

Deviations from (5) due to intermittency appear to be small (1%-2%). The departure was described by a non-linear behaviour, for example $\zeta_p^L = p/3-\mu_p$ (where the form of μ_p is determined by the selected intermittency model) (Frisch, 1995). Indeed a small difference should exist between $\zeta_{2,4}^T$ and $\zeta_{2,4}^L$ if the arguments of the logarithms (base r) are different from unity, that is if the ratios C_2 and C_4 are different from $(4/3)^{-1}$ and $(16/9)^{-1}$. However, these ratios are usually much more difficult to be derived from data than the scaling exponents themselves.

An extension of (5) to order p may lead one to conclude that the absolute magnitude of the scaling exponents of LSF and TSF are equal (Noullez *et al.*, 1997). However, this result is not supported by the majority of the available experimental and numerical data. In particular, there is only moderate support for (5), while there is much evidence to suggest that ζ_p^T is smaller than ζ_p^L (Boratav and Pelz 1997, Camussi et al. 1997): the difference is of the order of 20% for p=4 and 40% for p=8. The reason for this inequality clearly requires further investigation in the context of improving small-scale turbulence modelling.

There are several candidates to explain this inequality:

1. the anisotropy of the flow field;
2. the effect of Reynolds number;
3. the initial and boundary conditions;
4. inherently different intermittencies of LSF and TSF.

The first one seems to be the natural explanation for the observed differences: to this end the investigation of relations (1) and (3) can clarify the extent of this effect.

Relatively low Reynolds numbers, shorten the IR: this problem is particulary felt by TSF leading to bias in the evaluation of scaling exponents. Indeed, Pearson and Antonia (1999) found that the difference (ζ_p^L-ζ_p^T) decreased with increasing Re_λ (*i.e.* the Taylor microscale Reynolds number = $\lambda u'/\nu$). Nevertheless, the difference does not completely vanish even at quite high Reynolds numbers.

The effect of different initial and boundary conditions has not been investigated extensively: this leads to different injections of energy on the large scale which could affect also the IR. Some contribution should be expected especially when considering that different authors agree on a difference between scaling exponents of LSF and TSF but strongly disagree on the amount of this difference.

The possibility of a different source of intermittency between LSF and TSF has already been considered by some authors, in particular through possible differences in scaling of the locally averaged energy dissipation rate and the enstrophy. Recent DNS data (Boratav and Pelz, 1997) revealed that longitudinal velocity differences are mostly influenced by the former, whereas transverse difference depend on the latter. From measurements in grid turbulence (Antonia *et al.,* 1998) at moderate Reynolds numbers, the scaling exponents of LSF and TSF were inferred from those of the energy dissipation rate and of the enstrophy. The resulting differences of 2% for p=4 and 6% for p=8 are much smaller than measured differences. Incompressibility could be another source of inherent difference between LSF and TSF due to the different form of longitudinal and transverse correlations (Paret and Tabeling, 1998).

The aim of the paper is to estimate the difference ζ_p^L-ζ_p^T in a round jet where the anisotropy due to the large scale motion is likely to be more important than in grid turbulence. Moreover, the effect of different initial conditions is also considered by performing the experiment on two different jets. LSF and TSF are experimentally investigated in different regions of the jet flow by applying Taylor's hypothesis to data from Laser Doppler Anemometry (LDA) and Hot Wire Anemometry (HWA). Different Reynolds numbers are also tested.

EXPERIMENTAL SET UP

The far fields at the outlet of two round jets are investigated. Two different configurations were set-up: an air-jet (jet diameter = 55 mm, contraction ratio = 10, $Re=Ud/\nu\approx1.7\times10^5$, $Re_\lambda\approx545$) where measurements are made with HWA and a water-jet (jet diameter = 20 mm, contraction ratio =50, $Re=Ud/\nu\approx4.4\times10^4$, $Re_\lambda\approx500$) where measurements are made with LDA. Both measurements are performed at x/d=42 (x being the distance from the nozzle and d the jet diameter) where isotropy in the IR is expected. However, the ratio of measured *rms* axial and radial velocity fluctuations is still different from unity due to the anisotropic forcing by the large scales. It is about 1.15 for the air jet and 1.34 for the water jet. This difference is expected to reflect the effects of the different initial conditions. The Kolmogorov scales are $\eta\approx0.1$ mm and $v_K\approx$ 10 mm/s. More than 10^6 data samples were acquired.

Taylor's hypothesis is then applied to convert the temporal SF to a spatial SF. LSF is obtained from differences of the axial velocity component in time, whereas TSF is derived from differences of the radial velocity component in time. The LDA data are resampled using a zero-order interpolation to obtain equispaced data sets: at this rather high Reynolds number, this resampling does not affect the behaviour in the inertial range (Antonia *et al.* 1997, Romano *et al.* 1999).

In order to check that some of the disagreement between existing data, especially for ζ_3^T, may be due to the way the IR is identified, different procedures are considered:

1. evaluation of the IR as the range where the third-order LSF (p=3) increases linearly with r^* ($\zeta_3^L \approx r^*$, which is an exact result from the Navier-Stokes equations in isotropic turbulence): the scaling exponents of other ($p \neq 3$) structure functions are computed by fitting data in this range (also applies to transverse exponents);
2. evaluation of the IR as the range where the third-order LSF increases as r^*. The scaling exponents of the other longitudinal structure functions are computed by fitting data in this range, while the transverse scaling exponents in the range where $\zeta_3^T \approx r^*$ (this is only empirical and not based on Navier-Stokes equations);
3. evaluation of scaling exponents by using the Extended Self Similarity (ESS) (Benzi *et al.* 1993), that is by plotting the p^{th}-order structure function versus the 3^{rd}-order moment of the absolute increment and identifying the range where the behaviour is linear (on a log scale). ESS can be applied in method 1 (to both longitudinal and transverse) and 2 (only longitudinal).

Using procedure 1, the IR is evaluated to be from $r^*\approx50$ to $r^*\approx130$ for the HWA measurements in the air jet and from $r^*\approx20$ to $r^*\approx110$ for the LDA measurements on the water jet. Once the IR is established, the scaling exponents are computed using procedure 3. LDA measurements have been performed also at $Re\approx2\times10^5$ ($Re_\lambda\approx1100$).

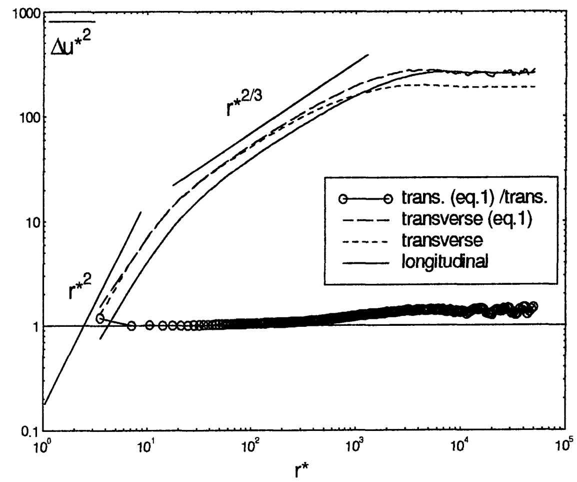

Figure 1. Second-order LSF and TSF from HWA measurements. The second-order TSF is also obtained from equation (1). Theoretical slopes in dissipative and inertial ranges are also shown.

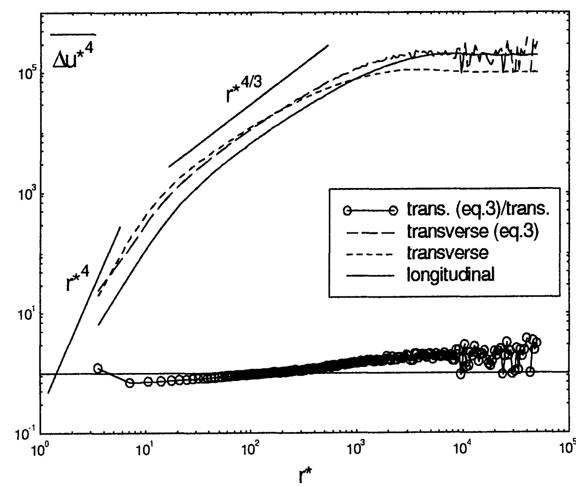

Figure 2. Fourth-order LSF and TSF from HWA measurements. The fourth-order TSF is also obtained from equation (3). Theoretical slopes in dissipative and inertial ranges are also shown.

RESULTS ON STRUCTURE FUNCTIONS

In figure 1, the second-order LSF and TSF are given for the HWA measurements in the air jet. In the same figure, the second-order TSF obtained using equation (1) is also given. The data on LSF seem to approach the theoretical predictions in the dissipative range and IR (without intermittency corrections), whereas the TSF shows a smaller slope in the IR. The TSF, from equation (1), approaches the measured TSF reasonably well, although with a slope in the IR which is close to that for the LSF.

In figure 2, the fourth-order LSF and TSF are given for the HWA measurements in the air jet together with the fourth-order TSF obtained using equation (3). Again, LSF reproduces the theoretical predictions in the dissipative range and IR (without intermittency), but TSF exhibits a slope significantly smaller than LSF in the IR. The TSF, obtained using equation (3), is close to the observed LSF.

From such functions, scaling exponents are computed using procedure 1. Second and fourth-order scaling exponents in the air and water jets are summarized in table 1, together with results obtained by Camussi *et al.* (1997) in a jet flow ($x/d \approx 20$, $Re_\lambda \approx 250$). For these data, the transverse exponents are evaluated using procedure 2. The differences between longitudinal and transverse exponents already reported by other authors are observed (about 15% for second-order and 20 % for the fourth-order). However, when transverse exponents are computed from the TSF obtained using equations (1) and (3), the difference is strongly reduced (within 5% for both second and fourth-orders). To this end, it should be noted that equations (1) and (3) are valid for locally isotropic turbulence and that intermittency effects in LSF are transferred to TSF. Therefore, our results indicate that a large part of the observed differences between scaling exponents of LSF and TSF is due to local anisotropies which affect the IR.

Some difference is also observed between exponents obtained from the air jet (HWA) and the water jet (LDA), the latter being always smaller than the former. The difference between longitudinal exponents is about 3%, compared to about 9% for the transverse exponents. In the water jet, the difference between longitudinal and transverse exponents is higher than in the air jet. The ratio between these differences is

$$\frac{\left(\varsigma^L_{2,4}-\varsigma^T_{2,4}\right)_w}{\left(\varsigma^L_{2,4}-\varsigma^T_{2,4}\right)_a}\approx 1.35-1.4,$$

which is also equal to the ratio between the mean squared velocity fluctuations due to the different initial conditions for the two jets

$$\frac{\left(\overline{u^2}/\overline{v^2}\right)_w}{\left(\overline{u^2}/\overline{v^2}\right)_a}\approx\left(\frac{1.34}{1.15}\right)^2=1.36.$$

Therefore, the two experimental conditions exhibit differences between the longitudinal and transverse exponents which are also dependent on the initial conditions through the ratio of fluctuations between the longitudinal (axial) and transverse velocity components.

Scaling exponents of p^{th}-order (up to 8) LSF and TSF are computed and given on figure 3. In the first part, the data are compared with those of Camussi *et al.* (1997).

	2^{nd}-order	4^{th}-order
ξ^L(HWA)	**0.713**	**1.303**
ξ^T (HWA)	**0.638**	**1.102**
ξ^T (HWA) (eq. 1, eq. 3)	**0.679**	**1.281**
ξ^L(LDA)	**0.711**	**1.261**
ξ^T (LDA)	**0.584**	**0.990**
ξ^T (LDA) (eq. 1, eq. 3)	**0.698**	**1.224**
ξ^L (Camussi *et al.*, 1997)	**0.70**	**1.27**
ξ^T (Camussi *et al.*, 1997)	**0.72**	**1.24**

Table 1. Second and fourth-order scaling exponents of LSF and TSF obtained from HWA and LDA measurements. Equations (1) and (3) are also used to evaluate the transverse exponents.

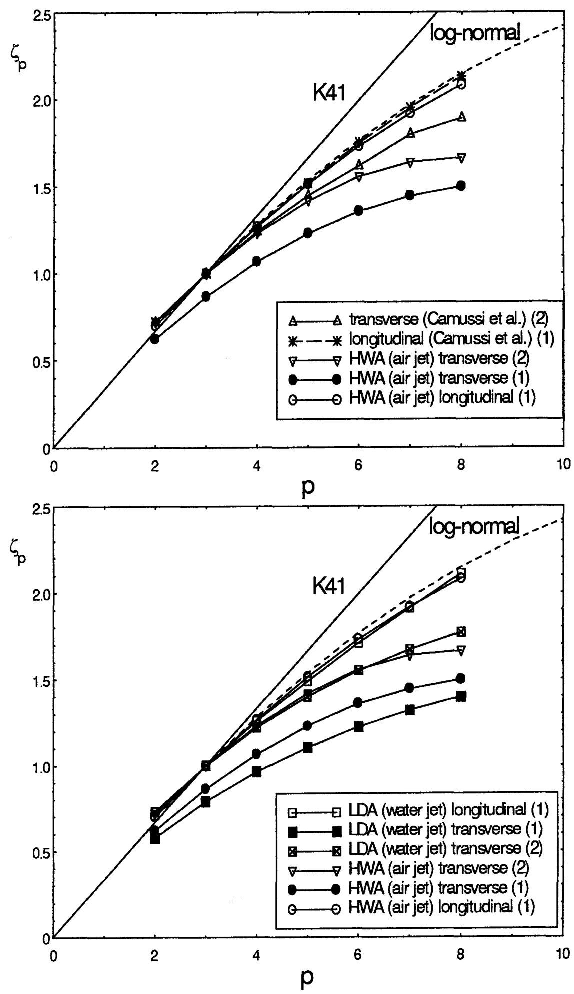

Figure 3. Scaling exponents of LSF and TSF obtained from HWA and LDA measurements. The number in brackets refers to the procedure used for evaluating the IR. The Camussi *et al.* (1997) data and the theoretical K41 and log-normal results are also shown.

In the second part of figure 3, the air jet (measured using HWA) and the water jet (using LDA) are compared. In the figure, both procedures 1 and 2 are used to evaluate the IR There is fairly good agreement between all longitudinal scaling exponents, irrespectively of initial conditions, Reynolds number and large scale anisotropy.

On the other hand, differences are observed in the transverse scaling exponents. The evaluation of scaling exponents for TSF, using an IR derived from third-order TSF (procedure 2), results in a decrease of the difference between longitudinal and transverse exponents (as in the data by Camussi *et al.*, 1997). The IR from third-order LSF and TSF is indicated on figure 4 for the LDA data (the SF is divided by r^*). This IR (constant using this type of plot) is between $r^*\approx20$ and $r^*\approx110$ for LSF, while the region where the third-order TSF is constant is much smaller (approximately between $r^*\approx15$ and $r^*\approx30$). When such a "transverse" IR, is used also for TSF, the present results (for both HWA and LDA) agree with those of Camussi *et al.* (1997). This highlights the fact that the evaluation of the IR is crucial for investigating TSF scaling exponents. However, we want to stress that procedure 2 is not supported by any theoretical argument and that the IR should be determined from the third-order LSF.

In the second part of figure 3, a comparison of scaling exponents between the two jets is shown. As for second and fourth-orders, the major difference is felt on transverse exponent. This difference changes with the order p, but slowly. It also depends on the procedure used to evaluate transverse exponents. This fact confirms that some of the existing difference in the evaluation of transverse exponents depends on how the IR is selected. However, it is somewhat surprising that this difference disappears when procedure 2 (without theoretical basis) is used.

The relative difference between longitudinal and transverse exponents is given in figure 5. For almost all data, it increases linearly with order p. The slope for the present data is about 0.03 (so a 3% increase for each step in p). This slope is slightly higher (about 0.035) when procedure 2 is used, and smaller (about 0.027) for procedure 1. On the other hand, a few differences are observed between slopes from the LDA and HWA data (less than 3%). The slope from Camussi et al. (1997) is definitely smaller than that for the present data (about 0.02). The data nearly collapse on a single curve when "erroneous" procedure 2 is used and the difference between longitudinal and transverse exponents is almost 5% for $p\leq4$ and within 10 % up to order $p=8$.

To evaluate the effect of the Reynolds number, the water jet measurements were repeated at a higher Re_λ (about 1100). In figure 6, the relative difference between longitudinal and transverse scaling exponents is given for different Reynolds numbers.

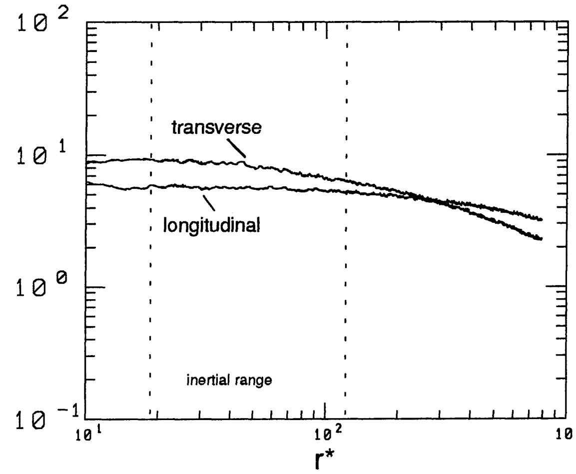

Figure 4. Third-order LSF and TSF divided by r^* from LDA measurements. The IR, determined from the third-order LSF, is also shown by vertical dotted lines.

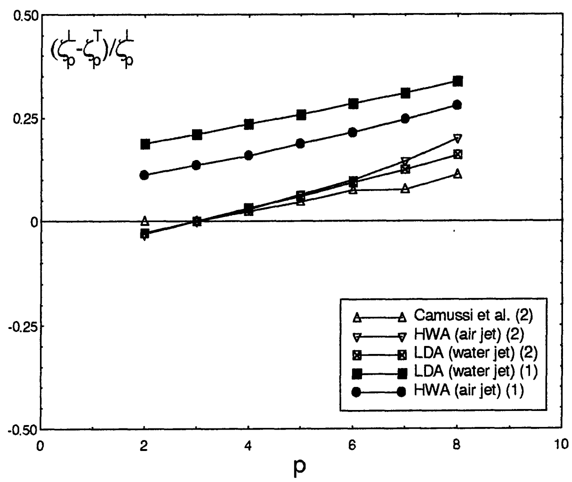

Figure 5. Relative difference between exponents of LSF and TSF. The number in brackets refers to the procedure used for evaluating the IR.

Except for the data of Camussi et al. (1997), whose initial conditions and turbulence fluctuation levels are not known, there is a clear trend which supports a reduction in the relative difference as the Reynolds number increases. However, even at the highest Reynolds number, the difference does not vanish if procedure 1 is used to evaluate the IR. On the other hand, using procedure 2, the difference vanishes and the transverse scaling exponents are even higher than the longitudinal. Further evidence for the decrease with increasing Re_λ in the relative difference between scaling exponents will be presented at this conference (Pearson and Antonia, 1999) in the context of a wide range of flows.

CONCLUSIONS

The differences between scaling exponents derived from longitudinal and transverse structure functions of velocity (LSF and TSF) are investigated experimentally in round jets. These differences are supposed to depend on the anisotropy of the velocity field through the:

- large scales (caused by different initial conditions);
- inertial range (IR) (caused by "local" deformations);
- small scales (through changes in Reynolds number).

These three sources are investigated in the present paper. In all tests, as reported by many authors, the longitudinal scaling exponents show negligible variations. Therefore, the attention is focused on transverse scaling exponents.

The determination of the IR is a crucial point for the evaluation of such exponents (and also accounts for some of the discrepancies between existing results). Although the use of the third-order LSF is the only one supported by the equations, surprisingly the use of third-order TSF allows the present data to collapse on a single curve, at least for small order p.

The effect of initial conditions is investigated by measuring SF in air and water jets with different longitudinal to transverse *rms* velocity fluctuation ratios. Transverse exponents differ from longitudinal exponents in both jets, although the difference is reduced as the ratio approaches unity.

The effect of anisotropy on the IR is determined by using relations, valid for a homogeneous, isotropic and incompressible flow, for second- and fourth-order SF which enable TSF to be obtained from LSF. In this way, the difference between longitudinal and transverse exponents is reduced within 5% in all tests. We stress that the fourth-order relation is only empirical and needs further testing.

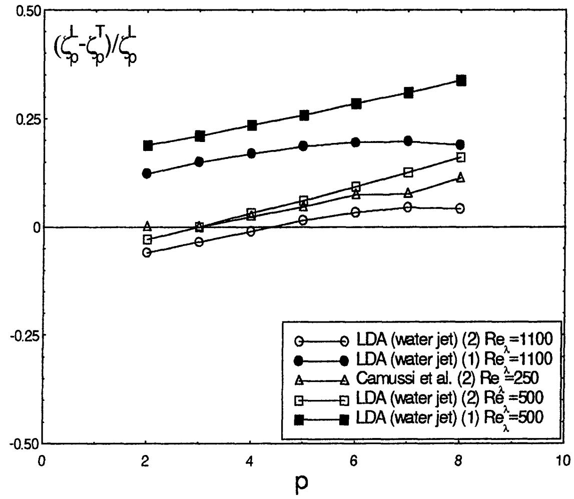

Figure 6. Relative difference between exponents of LSF and TSF at different Re_λ. The number in brackets refers to the procedure used for evaluating the IR.

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